q-Analogue Modified Laguerre Matrix Polynomials of Three Variables

Fadhl S.N. Alsarahi
Department of Mathematics, Faculty of Education, Yafea, Aden University, Yemen
DOI: https://doi.org/10.47372/uajnas.2021.n1.a12

Abstract

In this paper, the q-analogue modified Laguerre matrix polynomials of three variables are introduced as finite series and Some properties of these matrix polynomials are obtained.

Keywords: q-analogue modified Laguerre matrix polynomials; generating functions; recurrence relations.

1. Introduction

Matrix generalization of special functions has become important in the last two decades. The reason of importance have many motivations. For instance, using special matrix functions provides solutions for some physical problems. Also, special matrix functions are in connection with different matrix functions.

J´odar et al introduced Laguerre matrix polynomials in [11]. Some important and different properties of Laguerre matrix polynomials were investigated (see [1,2, 6,7, 9,11,18,20]).

Throughout this paper, for a matrix A in $C^{N\times N}$, its spectrum $\sigma(A)$ denotes the set of all eigenvalues of A.

The matrix analogues of Pochhammer symbol or shifted factorial is defined by [14]

$$(A)_n = A(A+I)(A+2I)\dots(A+(n-1)I),\ \ n\ge 1, (A)_0 = I\ , \tag{1.1}$$

where, $A\in C^{N\times N}$. The hypergeometric matrix function $F(A,B;C;z)$ is defined by [14]

$$F(A,B;C;z) = \sum_{n\ge 0}\frac{(A)_n(B)_n[(C)_n]^{-1}}{n!}z^n, \tag{1.2}$$

for matrices A,B,C in $C^{N\times N}$ such that $C+nI$ is invertible for all integers $n\ge 0$ and for $z<1$ (see [10]).

Furthermore, for a matrix A in $C^{N\times N}$, the authors exploited the following relation due to [10]:

$$(1-y)^{-A} = \sum_{n=0}^{\infty}\frac{(A)_n}{n!}y^n,\qquad |y|<1. \tag{1.3}$$

Also, for a matrix $A(k,n)$ in $C^{N\times N}$ for $n\ge 0$ and $k\ge 0$, the following relation is given by Defez and J´odar in [4]

$$\sum_{n=0}^{\infty}\sum_{k=0}^{\infty} A(k,n) = \sum_{n=0}^{\infty}\sum_{k=0}^{n} A(k,n-k). \tag{1.4}$$

$$\sum_{n=0}^{\infty}\sum_{k=0}^{n} A(k,n) = \sum_{n=0}^{\infty}\sum_{k=0}^{\infty} A(k,n+k). \tag{1.5}$$

We conclude this section by recalling the Laguerre matrix polynomials. Let A be a matrix in $C^{N\times N}$ such that $-k\notin\sigma(A)$ for every integer $k>0$ and λ be a complex number whose real part is positive. Then the Laguerre matrix polynomials $L_n^{(A,\lambda)}(x)$ are defined by [11]:

$$L_n^{(A,\lambda)}(x) = \sum_{k=0}^{n}\frac{(-1)^k(A+I)_n[(A+I)_k]^{-1}\lambda^k x^k}{k!(n-k)!}. \tag{1.6}$$

The generating function of Laguerre matrix polynomials is given in [11] by

$$(1-t)^{-(A+I)}exp\left(\frac{-\lambda xt}{1-t}\right) = \sum_{n=0}^{\infty}L_n^{(A,\lambda)}(x)\,t^n\ ,\ \ t\in\mathbb{C},\ \ |t|<1,\ \ x\in\mathbb{C},$$

and Rodrigues formula is

$$L_n^{(A,\lambda)}(x) = \frac{x^{-A}exp(\lambda x)}{n!}D^n[x^{A+nI}exp(-\lambda x)],\ \ n\ge 0. \tag{1.7}$$

Also, Laguerre matrix polynomials satisfy the three-term recurrence relation

$$(n+1)L_{n+1}^{(A,\lambda)}(x) + [\lambda xI-(A+(2n+1)I)]L_n^{(A,\lambda)}(x) + (A+nI)L_{n-1}^{(A,\lambda)}(x) = 0, \tag{1.8}$$

and second order matrix differential equation

$$\left[xD^2 + \left((A+I) - \lambda xI\right)D + \lambda nI\right]L_n^{(A,\lambda)}(x) = 0. \tag{1.9}$$

In [3], it is shown that an appropriate combination of methods, relevant to operational calculus and to matrix polynomials, can be a very useful tool to establish and treat a new class of two variable Laguerre matrix polynomials in the following form:

$$L_{n,m}^{(A,\lambda)}(x,y) = \sum_{s=0}^{n}\sum_{k=0}^{m} \frac{(-1)^{s+k}(A+I)_{n+m}[(A+I)_{s+k}]^{-1}(\lambda x)^s(\lambda y)^k}{s!(n-s)!k!(m-k)!},\ \{n,m\} \geq 0. \tag{1.10}$$

The generating relation for the matrix function $L_{n,m}^{(A,\lambda)}(x,y)$ is given by the formula:

$$(1-u-v)^{-(A+I)} exp\left(\frac{-\lambda(xu+yv)}{1-u-v}\right) = \sum_{n,m=0}^{\infty} L_{n,m}^{(A,\lambda)}(x,y)u^n v^m,$$

where $\{x,y,u,v\} \in \mathbb{C}$ and $|u+v| < 1$.

Recently, q-calculus has served as a bridge between mathematics and physics. Therefore, there is a significant increase of activity in the area of the q-calculus due to its applications in mathematics, statistics and physics.

Let the q-analogues of Pochhammer symbol or q-shifted factorial be defined by [8]

$$(a;q)_n = \begin{cases} 1 & , \quad n = 0 \\ \prod_{k=0}^{n-1}(1-aq^k), & n \in N \end{cases}, \tag{1.11}$$

also, $(a;q)_{n+k} = (a;q)_n(aq^n;q)_k$.

Now, the q-shifted factorials, where k and n are nonnegative integers [21]:

$$(a;q)_{n-k} = \frac{(a;q)_n}{(a^{-1}q^{1-n};q)_k}\left(-\frac{q}{a}\right)^k q^{\binom{k}{2}-nk},\ \ a \neq 0, k = 0,1,2,\dots,n. \tag{1.12}$$

The q-binomial coefficient (or Gaussian polynomial analogue to $\binom{n}{k}$ is defined by ([8] and [21])

$$\begin{bmatrix} n \\ k \end{bmatrix}_q = \frac{(q;q)_n}{(q;q)_{n-k}(q;q)_k} = \begin{bmatrix} n \\ n-k \end{bmatrix}_q, \tag{1.13}$$

$$\begin{bmatrix} \alpha \\ k \end{bmatrix}_q = \frac{(q^{-\alpha};q)_k}{(q;q)_k}(-q^\alpha)^k q^{-\binom{k}{2}} \quad, \alpha \in \mathbb{C}, k \in \mathrm{N}_0, \tag{1.14}$$

The q-analogue of the power (binomial) function $(x \pm y)^n$ ([17]) is given by

$$(x \pm y)^n \equiv (x \pm y)_n \equiv x^n\left(\pm {}^{y}\!/_{x}\,; q\right)_n = x^n \sum_{k=0}^{n} \begin{bmatrix} n \\ k \end{bmatrix}_q q^{\binom{k}{2}}\left(\pm {}^{y}\!/_{x}\right)^k. \tag{1.15}$$

The formulas for the q-difference D_q of a addition, a product and a quotient of functions are

$$D_q\left(\lambda f(x) + \mu g(x)\right) = \lambda D_q f(x) + \mu D_q g(x), \tag{1.16}$$

$$D_q\left(f(x).g(x)\right) = f(qx)D_q g(x) + g(x)D_q f(x), \tag{1.17}$$

$$D_q\left(\frac{f(x)}{g(x)}\right) = \frac{g(x)D_q f(x) - f(x)D_q g(x)}{g(x)g(qx)}, \quad g(x)g(qx) \neq 0. \tag{1.18}$$

The q-exponential function is defined by [21]:

$$e_q(x) = \sum_{n=0}^{\infty} \frac{x^n}{(q;q)_n}. \tag{1.19}$$

Moak (1981) introduced and studied the q-Laguerre polynomials [15]

$$L_n^{(\alpha)}(x;q) = \frac{(q^{\alpha+1};q)_n}{(q;q)_n}\sum_{k=0}^{n} \frac{(q^{-n};q)_k q^{k(k+1)/2}(q^{n+\alpha+1}x)^k}{[k]_q!(q^{\alpha+1};q)_k},\ \ \alpha > -1, n\epsilon N_0. \tag{1.20}$$

Fixed $0 < q < 1$ and $\alpha > 1$, the explicit form of the nth degree monic q-Laguerre polynomial reads ([15, 2.3])

$$L_n^{(\alpha)}(x;q) = \frac{(-1)^n(q^{\alpha+1};q)_n}{(1-q)^n}\sum_{k=0}^{n} \frac{(q^{-n};q)_k(1-q)^k q^{k(k+1)/2}}{(q^{\alpha+1};q)_k(q;q)_k q^{(n-k)(\alpha+n)}} x^k, \quad n\epsilon N_0. \tag{1.21}$$

Mohsen and Alsarahi [16] introduced q-analogue modified Laguerre polynomial of two variables by the following:

$$L_{n,m}^{(\alpha,\beta)}(x,y;q)=\frac{(q^m;q)_n(\beta y)^n}{q^{mn}(q;q)_n}\ {}_1\phi_1\ \left(q^{-n};q^m;q,-q^{m+1}\frac{\alpha x}{\beta y}\right). \quad (1.22)$$

where ${}_1\phi_1$ is the basic hypergeometric or q-hypergeometric function.

The generating relation for $L_{n,m}^{(A,\lambda)}(x,y)$ is given by the formula:

$$[1-\beta ty]_q^{-m}exp_q\left[\frac{-\alpha xt}{1-\beta ty}\right]=\sum_{n=0}^{\infty}\ L_{n,m}^{(\alpha,\beta)}(x,y;q)t^n. \quad (1.23)$$

2. q-Analogue Modified Laguerre Matrix Polynomials of Three Variables

In this section, we introduce the q-analogue modified Laguerre matrix polynomial of three variables by the following generating function:

$$[1-\beta z(u+v)]_q^{-(A+I)}exp_q\left(\frac{-\alpha(xu+yv)}{1-\beta z(u+v)}\right)=\sum_{n,m=0}^{\infty}L_{n,m}^{(A,\alpha,\beta)}(x,y,z;q)\ u^nv^m, \quad (2.1)$$

where $u,v,x,y,z\in\mathbb{C}$, $|z(u+v)|<1$.

Now, we get the series representation of the q-analogue modified Laguerre matrix polynomials in the form of the following theorem:

Theorem 2.1

Let us assume that A is a matrix in $C^{N\times N}$ and α,β be a complex number whose real part is positive, then the series representation of the q-analogue modified Laguerre matrix polynomials $L_{n,m}^{(A,\alpha,\beta)}(x,y,z;q)$ is given by:

$$L_{n,m}^{(A,\alpha,\beta)}(x,y,z;q)=\sum_{s=0}^{n}\sum_{k=0}^{m}\frac{q^{\binom{k}{2}+\binom{m-k}{2}+\binom{s+k}{2}-(n+m)(s+k)}}{(q;q)_s(q;q)_k}\frac{\left(q^{A+(s+k+1)I};q\right)_{n+m}}{(q;q)_{n-s}(q;q)_{m-k}}$$
$$\times\left[\left(q^{-(A+(n+m+s+k)I)};q\right)_{s+k}\right]^{-1}\left(\beta zq^{-(A+(s+k+1)I)}\right)^{n+m}\left(\frac{q\alpha x}{\beta z}\right)^s\left(\frac{q\alpha y}{\beta z}\right)^k. \quad (2.2)$$

Proof. Let us denote the left hand sides of (2.1) by W, then, by using of the q-exponential series (1.19), we get

$$W=\sum_{s=0}^{\infty}\frac{(-1)^s\alpha^s[xu+yv]_q^s}{(q;q)_s}[1-\beta z(u+v)]_q^{-(A+(s+1)I)}, \quad (2.3)$$

by using the relation (1.15) in (2.3), we get

$$W=\sum_{s=0}^{\infty}\frac{(-1)^s\alpha^s}{(q;q)_s}\sum_{k=0}^{s}\begin{bmatrix}s\\k\end{bmatrix}_q q^{\binom{k}{2}}(xu)^{s-k}(yv)^k$$
$$\times\sum_{n=0}^{\infty}\begin{bmatrix}-(A+(s+1)I)\\n\end{bmatrix}_q q^{\binom{n}{2}}\ [-\beta z(u+v)]_q^n, \quad (2.4)$$

applying relations (1.13), (1.14) and (1.15) on (2.4), we obtain

$$W=\sum_{s=0}^{\infty}\sum_{k=0}^{s}\frac{(-1)^s\alpha^sq^{\binom{k}{2}}(xu)^{s-k}(yv)^k}{(q;q)_{s-k}(q;q)_k}\sum_{n=0}^{\infty}\frac{\left(q^{A+(s+1)I};q\right)_n}{(q;q)_n}\left(-q^{-(A+(s+1)I)}\right)^n$$
$$\times\sum_{m=0}^{n}\frac{(q;q)_n}{(q;q)_{n-m}(q;q)_m}q^{\binom{m}{2}}(-\beta z)^n(u)^{n-m}(v)^m, \quad (2.5)$$

which on using relation (1.4), gives

$$W=\sum_{n,m=0}^{\infty}\sum_{s,k=0}^{\infty}\frac{(-1)^{s+k}\alpha^{s+k}q^{\binom{k}{2}+\binom{m}{2}}}{(q;q)_s(q;q)_k}\frac{\left(q^{A+(s+k+1)I};q\right)_{n+m}}{(q;q)_n(q;q)_m}$$
$$\times\left(\beta zq^{-(A+(s+k+1)I)}\right)^{n+m}x^sy^k(u)^{n+s}(v)^{m+k}, \quad (2.6)$$

using the relation (1.5), we find

$$W = \sum_{n,m=0}^{\infty}\sum_{s=0}^{n}\sum_{k=0}^{m} \frac{(-1)^{s+k}\alpha^{s+k}q^{\binom{k}{2}+\binom{m-k}{2}}}{(q;q)_s(q;q)_k}\frac{\left(q^{A+(s+k+1)I};q\right)_{n+m-(s+k)}}{(q;q)_{n-s}(q;q)_{m-k}} \times \left(\beta z q^{-(A+(s+k+1)I)}\right)^{n+m-(s+k)} x^s y^k u^n v^m, \quad (2.7)$$

applying relation (1.12) on (2.7), we obtain

$$W = \sum_{n,m=0}^{\infty}\sum_{s=0}^{n}\sum_{k=0}^{m} \frac{(-1)^{s+k}q^{\binom{k}{2}+\binom{m-k}{2}}}{(q;q)_s(q;q)_k}\frac{\left(q^{A+(s+k+1)I};q\right)_{n+m}}{(q;q)_{n-s}(q;q)_{m-k}} \times \left[\left(q^{-(A+(n+m+s+k)I)};q\right)_{s+k}\right]^{-1}\left(-q^{-(A+(s+k)I)}\right)^{s+k} q^{\binom{s+k}{2}-(n+m)(s+k)} \left(\beta z q^{-(A+(s+k+1)I)}\right)^{n+m-(s+k)}(\alpha x)^s(\alpha y)^k u^n v^m, \quad (2.8)$$

by equating the coefficients of $u^n v^m$, we obtain the relation (2.2).

Next, we derive some recurrence relations for the polynomials $L_{n,m}^{(A,\alpha,\beta)}(x,y,z;q)$ in the form of the following theorems:

Theorem 2.2

The q-analogue modified Laguerre matrix polynomials of three variables $L_{n,m}^{(A,\alpha,\beta)}(x,y,z;q)$ satisfy the following relations:

$$\frac{\partial^r}{\partial x^r}L_{n,m}^{(A,\alpha,\beta)}(x,y,z;q) = (-\alpha)^r\sum_{s=0}^{n-r}\sum_{k=0}^{m} \frac{q^{\binom{k}{2}+\binom{m-k}{2}+\binom{s+k}{2}-(n-r+m)(s+k)}}{(q;q)_s(q;q)_k}\frac{\left(q^{A+(s+k+1)I};q\right)_{(n-r)+m}}{(q;q)_{(n-r)-s}(q;q)_{m-k}} \times \left[\left(q^{-(A+(n-r+m+s+k+1)I)};q\right)_{s+k}\right]^{-1}\left(\beta z q^{-(A+(s+k+2)I)}\right)^{(n-r)+m}\left(\frac{q\alpha x}{\beta z}\right)^s\left(\frac{q\alpha y}{\beta z}\right)^k, \quad (2.8)$$

and

$$\frac{\partial^r}{\partial y^r}L_{n,m}^{(A,\alpha,\beta)}(x,y,z;q) = (-\alpha)^r\sum_{s=0}^{n}\sum_{k=0}^{m-r} \frac{q^{\binom{k}{2}+\binom{m-k-r}{2}+\binom{s+k}{2}-(n+m-r)(s+k)}}{(q;q)_s(q;q)_k}\frac{\left(q^{A+(s+k+1)I};q\right)_{n+(m-r)}}{(q;q)_{n-s}(q;q)_{(m-r)-k}} \times \left[\left(q^{-(A+(n+m-r+s+k+1)I)};q\right)_{s+k}\right]^{-1}\left(\beta z q^{-(A+(s+k+2)I)}\right)^{n+(m-r)}\left(\frac{q\alpha x}{\beta z}\right)^s\left(\frac{q\alpha y}{\beta z}\right)^k. \quad (2.9)$$

Proof. Differentiating both sides of (2.1) with respect to x, we get

$$\sum_{n,m=0}^{\infty}\frac{\partial}{\partial x}L_{n,m}^{(A,\alpha,\beta)}(x,y,z;q)\,u^n v^m = -\alpha u\sum_{s=0}^{\infty}\frac{(-1)^s\alpha^s[xu+yv]_q^s}{(q;q)_s}[1-\beta z(u+v)]_q^{-(A+(s+2)I)}, \quad (2.10)$$

applying relation (1.15) in (2.10), we get

$$\sum_{n,m=0}^{\infty}\frac{\partial}{\partial x}L_{n,m}^{(A,\alpha,\beta)}(x,y,z;q)\,u^n v^m = -\alpha u\sum_{s=0}^{\infty}\frac{(-1)^s\alpha^s}{(q;q)_s}\sum_{k=0}^{s}\begin{bmatrix} s \\ k\end{bmatrix}_q q^{\binom{k}{2}}(xu)^{s-k}(yv)^k \times \sum_{n=0}^{\infty}\begin{bmatrix} -(A+(s+2)I) \\ n\end{bmatrix}_q q^{\binom{n}{2}}[-\beta z(u+v)]_q^n, \quad (2.11)$$

by using the relations (1.13), (1.14) and (1.15) in (2.11), we obtain

$$\sum_{n,m=0}^{\infty}\frac{\partial}{\partial x}L_{n,m}^{(A,\alpha,\beta)}(x,y,z;q)\,u^n v^m$$
$$=-\alpha u\sum_{s=0}^{\infty}\sum_{k=0}^{s}\frac{(-1)^s\alpha^s q^{\binom{k}{2}}(xu)^{s-k}(yv)^k}{(q;q)_{s-k}(q;q)_k}\sum_{n=0}^{\infty}\frac{\left(q^{A+(s+2)I};q\right)_n}{(q;q)_n}$$
$$\times\left(-q^{-(A+(s+2)I)}\right)^n\sum_{m=0}^{n}\frac{(q;q)_n}{(q;q)_{n-m}(q;q)_m}q^{\binom{m}{2}}(-\beta z)^n(u)^{n-m}(v)^m, \quad (2.12)$$

using the relation (1.4), we find

$$\sum_{n,m=0}^{\infty}\frac{\partial}{\partial x}L_{n,m}^{(A,\alpha,\beta)}(x,y,z;q)\,u^n v^m$$
$$=-\alpha\sum_{n,m=0}^{\infty}\sum_{s,k=0}^{\infty}\frac{(-1)^{s+k}\alpha^{s+k}q^{\binom{k}{2}+\binom{m}{2}}}{(q;q)_s(q;q)_k}\frac{\left(q^{A+(s+k+2)I};q\right)_{n+m}}{(q;q)_n(q;q)_m}$$
$$\times\left(\beta z q^{-(A+(s+k+2)I)}\right)^{n+m}x^s y^k(u)^{n+s+1}(v)^{m+k}, \quad (2.13)$$

which on using relation (1.5), gives

$$\sum_{n,m=0}^{\infty}\frac{\partial}{\partial x}L_{n,m}^{(A,\alpha,\beta)}(x,y,z;q)\,u^n v^m$$
$$=-\alpha\sum_{n,m=0}^{\infty}\sum_{s=0}^{n-1}\sum_{k=0}^{m}\frac{(-1)^{s+k}\alpha^{s+k}q^{\binom{k}{2}+\binom{m-k}{2}}}{(q;q)_s(q;q)_k}\frac{\left(q^{A+(s+k+2)I};q\right)_{(n-1)+m-(s+k)}}{(q;q)_{(n-1)-s}(q;q)_{m-k}}$$
$$\times\left(\beta z q^{-(A+(s+k+2)I)}\right)^{(n-1)+m-(s+k)}x^s y^k u^n v^m, \quad (2.14)$$

applying relation (1.12) in (2.14), we obtain

$$\sum_{n,m=0}^{\infty}\frac{\partial}{\partial x}L_{n,m}^{(A,\alpha,\beta)}(x,y,z;q)\,u^n v^m$$
$$=-\alpha\sum_{n,m=0}^{\infty}\sum_{s=0}^{n-1}\sum_{k=0}^{m}\frac{(-1)^{s+k}q^{\binom{k}{2}+\binom{m-k}{2}}}{(q;q)_s(q;q)_k}\frac{\left(q^{A+(s+k+2)I};q\right)_{(n-1)+m}}{(q;q)_{(n-1)-s}(q;q)_{m-k}}$$
$$\times\left[\left(q^{-(A+(n+m+s+k+1)I)};q\right)_{s+k}\right]^{-1}\left(-q^{-(A+(s+k+1)I)}\right)^{s+k}q^{\binom{s+k}{2}-(n-1+m)(s+k)}$$
$$\left(\beta z q^{-(A+(s+k+2)I)}\right)^{(n-1)+m-(s+k)}(\alpha x)^s(\alpha y)^k u^n v^m, \quad (2.15)$$

by equating the coefficients of $u^n v^m$, we obtain

$$\frac{\partial}{\partial x}L_{n,m}^{(A,\alpha,\beta)}(x,y,z;q)$$
$$=-\alpha\sum_{s=0}^{n-1}\sum_{k=0}^{m}\frac{q^{\binom{k}{2}+\binom{m-k}{2}+\binom{s+k}{2}-(n-1+m)(s+k)}}{(q;q)_s(q;q)_k}\frac{\left(q^{A+(s+k+1)I};q\right)_{(n-1)+m}}{(q;q)_{(n-1)-s}(q;q)_{m-k}}$$
$$\times\left[\left(q^{-(A+(n+m+s+k+1)I)};q\right)_{s+k}\right]^{-1}\left(\beta z q^{-(A+(s+k+2)I)}\right)^{(n-1)+m}\left(\frac{q\alpha x}{\beta z}\right)^s\left(\frac{q\alpha y}{\beta z}\right)^k.$$

Thus, by same manner as above, we can obtain

$$\frac{\partial^2}{\partial x^2}L_{n,m}^{(A,\alpha,\beta)}(x,y;q)$$
$$=\alpha^2\sum_{s=0}^{n-2}\sum_{k=0}^{m}\frac{q^{\binom{k}{2}+\binom{m-k}{2}+\binom{s+k}{2}-(n-2+m)(s+k)}}{(q;q)_s(q;q)_k}\frac{\left(q^{A+(s+k+1)I};q\right)_{(n-2)+m}}{(q;q)_{(n-2)-s}(q;q)_{m-k}}$$
$$\times\left[\left(q^{-(A+(n+m+s+k)I)};q\right)_{s+k}\right]^{-1}\left(\beta z q^{-(A+(s+k+2)I)}\right)^{(n-2)+m}\left(\frac{q\alpha x}{\beta z}\right)^s\left(\frac{q\alpha y}{\beta z}\right)^k.$$

Hence, by continuing the above steps, we get the required relation (2.8).

Similarly, differentiating (2.1), with respect to y ,we get relation (2.9).

Theorem 2.3

The q-analogue modified Laguerre matrix polynomials of three variables $L_{n,m}^{(A,\alpha,\beta)}(x,y,z;q)$ satisfy the following relations:

$$\begin{aligned}
&[n+1]_q L_{n+1,m}^{(A,\alpha,\beta)}(x,y,z;q)\\
&= -\alpha x \sum_{s=0}^{n}\sum_{k=0}^{m}\sum_{t=0}^{n-s}\sum_{p=0}^{m-k} \frac{(-1)^{s+k} q^{\binom{k}{2}+\binom{m-k-p}{2}+\binom{p}{2}+t} (q^{A+2I};q)_{t+p}}{(q;q)_s (q;q)_k (q;q)_t (q;q)_p}\\
&\times \frac{\left(q^{A+(s+k+1)I};q\right)_{n+m}\left[\left(q^{-(A+(n+m+s+k)I};q\right)_{(s+t+k+p)}\right]^{-1}}{(q;q)_{n-s-t}(q;q)_{m-k-p}}\left(q^{-(A+I)}\right)^{t+p}\\
&\times \left(\beta z q^{-(A+(s+k+1)I)}\right)^{n+m} q^{\binom{(s+t+k+p)}{2}-(n+m)((s+t+k+p))}\left(\frac{q\alpha x}{\beta z}\right)^{s}\left(\frac{q\alpha y}{\beta z}\right)^{k}\\
&+\alpha(x-y)\sum_{s=0}^{n}\sum_{k=0}^{m-1}\sum_{t=0}^{n-s}\sum_{p=0}^{m-k-1} \frac{(-1)^{s+k} q^{\binom{k}{2}+\binom{m-k-p}{2}+\binom{p}{2}+t+1}}{(q;q)_s (q;q)_k (q;q)_t (q;q)_p}\\
&\times \frac{(q^{A+2I};q)_{t+p}\left(q^{A+(s+k+1)I};q\right)_{n+m}\left[\left(q^{-(A+(n+m+s+k)I};q\right)_{(s+t+k+p+1)}\right]^{-1}}{(q;q)_{n-s-t}(q;q)_{m-k-p-1}}\\
&\times \left(\beta z q^{-(A+(s+k+1)I)}\right)^{n+m} q^{\binom{(s+t+k+p+1)}{2}-(n+m)((s+t+k+p+1))}\\
&\times \left(q^{-(A+I)}\right)^{t+p}\left(\frac{q\alpha x}{\beta z}\right)^{s}\left(\frac{q\alpha y}{\beta z}\right)^{k}\\
&+\beta z(A+I)\sum_{s=0}^{n}\sum_{k=0}^{m} \frac{(-1)^{s+k} q^{\binom{k}{2}+\binom{m-k}{2}}}{(q;q)_s (q;q)_k}\frac{\left(q^{A+(s+k+2)I};q\right)_{n+m}}{(q;q)_{n-s}(q;q)_{m-k}} q^{\binom{s+k}{2}-(n+m)(s+k)}\\
&\times \left[\left(q^{A+(s+k+1)I};q\right)_{(s+k)}\right]^{-1}\left(\beta z q^{-(A+(s+k+2)I)}\right)^{n+m}\left(\frac{q\alpha x}{\beta z}\right)^{s}\left(\frac{q\alpha y}{\beta z}\right)^{k}, \qquad (2.16)
\end{aligned}$$

and

$$\begin{aligned}
&[m+1]_q L_{n,m+1}^{(A,\alpha,\beta)}(x,y,z;q)\\
&= -\alpha x \sum_{s=0}^{n}\sum_{k=0}^{m}\sum_{t=0}^{n-s}\sum_{p=0}^{m-k} \frac{(-1)^{s+k} q^{\binom{k}{2}+\binom{m-k-p}{2}+\binom{p}{2}+t} (q^{A+2I};q)_{t+p}}{(q;q)_s (q;q)_k (q;q)_t (q;q)_p}\\
&\times \frac{\left(q^{A+(s+k+1)I};q\right)_{n+m}\left[\left(q^{-(A+(n+m+s+k)I};q\right)_{(s+t+k+p)}\right]^{-1}}{(q;q)_{n-s-t}(q;q)_{m-k-p}}\left(q^{-(A+I)}\right)^{t+p}\\
&\times \left(\beta z q^{-(A+(s+k+1)I)}\right)^{n+m} q^{\binom{(s+t+k+p)}{2}-(n+m)((s+t+k+p))}\left(\frac{q\alpha x}{\beta z}\right)^{s}\left(\frac{q\alpha y}{\beta z}\right)^{k}\\
&+\alpha(y-x)\sum_{s=0}^{n-1}\sum_{k=0}^{m}\sum_{t=0}^{n-s-1}\sum_{p=0}^{m-k} \frac{(-1)^{s+k} q^{\binom{k}{2}+\binom{m-k-p}{2}+\binom{p}{2}+t+1}}{(q;q)_s (q;q)_k (q;q)_t (q;q)_p}\\
&\times \frac{(q^{A+2I};q)_{t+p}\left(q^{A+(s+k+1)I};q\right)_{n+m}\left[\left(q^{-(A+(n+m+s+k)I};q\right)_{(s+t+k+p+1)}\right]^{-1}}{(q;q)_{n-s-t-1}(q;q)_{m-k-p}}\\
&\times \left(\beta z q^{-(A+(s+k+1)I)}\right)^{n+m} q^{\binom{(s+t+k+p+1)}{2}-(n+m)((s+t+k+p+1))}
\end{aligned}$$

$$\times \left(q^{-(A+I)}\right)^{t+p} \left(\frac{q\alpha x}{\beta z}\right)^{s} \left(\frac{q\alpha y}{\beta z}\right)^{k}$$

$$+\beta z(A+I)\sum_{s=0}^{n}\sum_{k=0}^{m} \frac{(-1)^{s+k} q^{\binom{k}{2}+\binom{m-k}{2}}}{(q;q)_s (q;q)_k} \frac{\left(q^{A+(s+k+2)I};q\right)_{n+m}}{(q;q)_{n-s}(q;q)_{m-k}} q^{\binom{s+k}{2}-(n+m)(s+k)}$$

$$\times \left[\left(q^{A+(s+k+1)I};q\right)_{(s+k)}\right]^{-1} \left(\beta z q^{-(A+(s+k+2)I)}\right)^{n+m} \left(\frac{q\alpha x}{\beta z}\right)^{s} \left(\frac{q\alpha y}{\beta z}\right)^{k}. \qquad (2.17)$$

Proof. Differentiating the both sides of (2.1) , with respect to u and using relations (1.17),(1.18) , we get

$$\sum_{n,m=0}^{\infty} [n]_q L_{n,m}^{(A,\alpha,\beta)}(x,y,z;q)\, u^{n-1} v^m$$

$$= [1-\beta z(qu+v)]_q^{-(A+I)} \left(\frac{-\alpha x+\alpha\beta z v(x-y)}{(1-\beta z(u+v))(1-\beta z(qu+v))}\right) exp_q\left(\frac{-\alpha(xu+yv)}{1-\beta z(u+v)}\right)$$

$$+\beta z(A+I)[1-\beta z(u+v)]_q^{-(A+2I)} exp_q\left(\frac{-\alpha(xu+yv)}{1-\beta z(u+v)}\right), \qquad (2.18)$$

applying relation (1.19) in (2.18), we get

$$\sum_{n,m=0}^{\infty} [n+1]_q L_{n+1,m}^{(A,\alpha,\beta)}(x,y,z;q)\, u^n\, v^m = \left(-\alpha x+\alpha\beta z v(x-y)\right)$$

$$\times [1-\beta z(qu+v)]_q^{-(A+2I)} \sum_{s=0}^{\infty} \frac{(-1)^s \alpha^s [xu+yv]_q^s}{(q;q)_s} [1-\beta z(u+v)]_q^{-(A+(s+1)I)}$$

$$+\beta z(A+I)\sum_{s=0}^{\infty} \frac{(-1)^s\alpha^s[xu+yv]_q^s}{(q;q)_s}[1-\beta z(u+v)]_q^{-(A+(s+2)I)}, \qquad (2.19)$$

by using the relation (1.15) in (2.19), we get

$$\sum_{n,m=0}^{\infty} [n+1]_q L_{n+1,m}^{(A,\alpha,\beta)}(x,y,z;q)\, u^n\, v^m = \left(-\alpha x+\alpha\beta z v(x-y)\right)$$

$$\times \sum_{t=0}^{\infty} \begin{bmatrix} -(A+2I) \\ t \end{bmatrix}_q q^{\binom{t}{2}} [-\beta z(qu+v)]_q^t \sum_{s=0}^{\infty} \frac{(-1)^s\alpha^s}{(q;q)_s}$$

$$\times \sum_{k=0}^{s} \begin{bmatrix} s \\ k \end{bmatrix}_q q^{\binom{k}{2}} (xu)^{s-k}(yv)^k \sum_{n=0}^{\infty} \begin{bmatrix} -(A+(s+1)I) \\ n \end{bmatrix}_q q^{\binom{n}{2}} [-\beta z(u+v)]_q^n$$

$$+\beta z(A+I)\sum_{s=0}^{\infty} \frac{(-1)^s\alpha^s}{(q;q)_s} \sum_{k=0}^{s} \begin{bmatrix} s \\ k \end{bmatrix}_q q^{\binom{k}{2}}(xu)^{s-k}(yv)^k$$

$$\times \sum_{n=0}^{\infty} \begin{bmatrix} -(A+(s+2)I) \\ n \end{bmatrix}_q q^{\binom{n}{2}} [-\beta z(u+v)]_q^n, \qquad (2.20)$$

which using relations (1.13) and (1.14) in (2.20), gives

$$\sum_{n,m=0}^{\infty} [n+1]_q L_{n+1,m}^{(A,\alpha,\beta)}(x,y;q)\, u^n\, v^m = \left(-\alpha x+\alpha\beta z v(x-y)\right)$$

$$\times \sum_{t=0}^{\infty} \frac{(q^{A+2I};q)_t}{(q;q)_t} \left(-q^{-(A+2I)}\right)^t \sum_{p=0}^{t} \frac{(q;q)_t q^{\binom{p}{2}}(-\beta z)^t}{(q;q)_{t-p}(q;q)_p} (qu)^{t-p}(v)^p$$

$$\times \sum_{s=0}^{\infty}\sum_{k=0}^{s} \frac{(-1)^s\alpha^s q^{\binom{k}{2}}(xu)^{s-k}(yv)^k}{(q;q)_{s-k}(q;q)_k} \sum_{n=0}^{\infty} \frac{\left(q^{A+(s+1)I};q\right)_n}{(q;q)_n} \left(-q^{-(A+(s+1)I)}\right)^n$$

$$\times \sum_{m=0}^{n} \frac{(q;q)_n}{(q;q)_{n-m}(q;q)_m} q^{\binom{m}{2}} (-\beta z)^n (u)^{n-m} (v)^m$$
$$+ \beta z(A+I) \sum_{s=0}^{\infty}\sum_{k=0}^{s} \frac{(-1)^s \alpha^s q^{\binom{k}{2}} (xu)^{s-k} (yv)^k}{(q;q)_{s-k}(q;q)_k} \sum_{n=0}^{\infty} \frac{\left(q^{A+(s+2)I};q\right)_n}{(q;q)_n}$$
$$\times \left(-q^{-(A+(s+2)I)}\right)^n \sum_{m=0}^{n} \frac{(q;q)_n}{(q;q)_{n-m}(q;q)_m} q^{\binom{m}{2}} (-\beta z)^n (u)^{n-m} (v)^m, \qquad (2.21)$$

which using relation (1.5) in (2.21), gives

$$\sum_{n,m=0}^{\infty} [n+1]_q L_{n+1,m}^{(A,\alpha,\beta)}(x,y,z;q)\, u^n\, v^m = \left(-\alpha x + \alpha\beta z v(x-y)\right)$$
$$\times \sum_{t,p=0}^{\infty} \frac{q^{\binom{p}{2}} (q^{A+2I};q)_{t+p}}{(q;q)_t (q;q)_p} \left(\beta z q^{-(A+2I)}\right)^{t+p} (qu)^t (v)^p$$
$$\times \sum_{n,m=0}^{\infty}\sum_{s,k=0}^{\infty} \frac{(-1)^{s+k} q^{\binom{k}{2}+\binom{m}{2}}}{(q;q)_s(q;q)_k} \frac{\left(q^{A+(s+k+1)I};q\right)_{n+m}}{(q;q)_n(q;q)_m}$$
$$\times \left(\beta z q^{-(A+(s+k+1)I)}\right)^{n+m} (\alpha x)^s (\alpha y)^k (u)^{n+s} (v)^{m+k}$$
$$+\beta z(A+I) \sum_{n,m=0}^{\infty}\sum_{s,k=0}^{\infty} \frac{(-1)^{s+k} q^{\binom{k}{2}+\binom{m}{2}}}{(q;q)_s(q;q)_k} \frac{\left(q^{A+(s+k+2)I};q\right)_{n+m}}{(q;q)_n(q;q)_m}$$
$$\times \left(\beta z q^{-(A+(s+k+2)I)}\right)^{n+m} (\alpha x)^s (\alpha y)^k (u)^{n+s} (v)^{m+k}, \qquad (2.22)$$

by using relation (1.4) in (2.22), we get

$$\sum_{n,m=0}^{\infty} [n+1]_q L_{n+1,m}^{(A,\alpha,\beta)}(x,y,z;q)\, u^n\, v^m$$
$$= -\alpha x \sum_{n,m=0}^{\infty} \sum_{s=0}^{n}\sum_{k=0}^{m}\sum_{t=0}^{n-s}\sum_{p=0}^{m-k} \frac{(-1)^{s+k} q^{\binom{k}{2}+\binom{m-k-p}{2}+\binom{p}{2}+t} (q^{A+2I};q)_{t+p}}{(q;q)_s(q;q)_k(q;q)_t(q;q)_p}$$
$$\times \frac{\left(q^{A+(s+k+1)I};q\right)_{n+m-(s+t+k+p)}}{(q;q)_{n-s-t}(q;q)_{m-k-p}} \left(\beta z q^{-(A+(s+k+1)I)}\right)^{n+m-(s+t+k+p)}$$
$$\times \left(\beta z q^{-(A+2I)}\right)^{t+p} (\alpha x)^s (\alpha y)^k (u)^n (v)^m$$
$$+\alpha\beta z(x-y) \sum_{n,m=0}^{\infty} \sum_{s=0}^{n}\sum_{k=0}^{m-1}\sum_{t=0}^{n-s}\sum_{p=0}^{m-k-1} \frac{(-1)^{s+k} q^{\binom{k}{2}+\binom{m-k-p}{2}+\binom{p}{2}+t}}{(q;q)_s(q;q)_k(q;q)_t(q;q)_p}$$
$$\times \frac{(q^{A+2I};q)_{t+p} \left(q^{A+(s+k+1)I};q\right)_{n+m-(s+t+k+p+1)}}{(q;q)_{n-s-t}(q;q)_{m-k-p-1}}$$
$$\times \left(\beta z q^{-(A+(s+k+1)I)}\right)^{n+m-(s+t+k+p+1)} \left(\beta q^{-(A+2I)}\right)^{t+p} (\alpha x)^s (\alpha y)^k (u)^n (v)^m$$
$$+\beta z(A+I) \sum_{n,m=0}^{\infty}\sum_{s=0}^{n}\sum_{k=0}^{m} \frac{(-1)^{s+k} q^{\binom{k}{2}+\binom{m-k}{2}}}{(q;q)_s(q;q)_k} \frac{\left(q^{A+(s+k+2)I};q\right)_{n+m-(s+k)}}{(q;q)_{n-s}(q;q)_{m-k}}$$
$$\times \left(\beta z q^{-(A+(s+k+2)I)}\right)^{n+m-(s+k)} (\alpha x)^s (\alpha y)^k (u)^n (v)^m, \qquad (2.23)$$

applying relation (1.12) in (2.14), we obtain

$$\sum_{n,m=0}^{\infty}[n+1]_q L_{n+1,m}^{(A,\alpha,\beta)}(x,y,z;q)\,u^n\,v^m$$

$$=-\alpha x\sum_{n,m=0}^{\infty}\sum_{s=0}^{n}\sum_{k=0}^{m}\sum_{t=0}^{n-s}\sum_{p=0}^{m-k}\frac{(-1)^{s+k}q^{\binom{k}{2}+\binom{m-k-p}{2}+\binom{p}{2}+t}(q^{A+2I};q)_{t+p}}{(q;q)_s(q;q)_k(q;q)_t(q;q)_p}$$

$$\times\frac{\left(q^{A+(s+k+1)I};q\right)_{n+m}\left[\left(q^{-(A+(n+m+s+k)I};q\right)_{(s+t+k+p)}\right]^{-1}}{(q;q)_{n-s-t}(q;q)_{m-k-p}}$$

$$\times\left(q^{-(A+(s+k)I}\right)^{(s+t+k+p)}\left(\beta zq^{-(A+(s+k+1)I)}\right)^{n+m-(s+t+k+p)}$$

$$\times q^{\binom{(s+t+k+p)}{2}-(n+m)((s+t+k+p))}\left(\beta zq^{-(A+2I)}\right)^{t+p}(\alpha x)^s(\alpha y)^k(u)^n(v)^m$$

$$+\alpha\beta z(x-y)\sum_{n,m=0}^{\infty}\sum_{s=0}^{n}\sum_{k=0}^{m-1}\sum_{t=0}^{n-s}\sum_{p=0}^{m-k-1}\frac{(-1)^{s+k}q^{\binom{k}{2}+\binom{m-k-p}{2}+\binom{p}{2}+t}}{(q;q)_s(q;q)_k(q;q)_t(q;q)_p}$$

$$\times\frac{(q^{A+2I};q)_{t+p}\left(q^{A+(s+k+1)I};q\right)_{n+m}\left[\left(q^{-(A+(n+m+s+k)I};q\right)_{(s+t+k+p+1)}\right]^{-1}}{(q;q)_{n-s-t}(q;q)_{m-k-p-1}}$$

$$\times\left(q^{-(A+(s+k)I}\right)^{(s+t+k+p+1)}\left(\beta zq^{-(A+(s+k+1)I)}\right)^{n+m-(s+t+k+p+1)}$$

$$\times q^{\binom{(s+t+k+p+1)}{2}-(n+m)((s+t+k+p+1))}\left(\beta zq^{-(A+2I)}\right)^{t+p}(\alpha x)^s(\alpha y)^k(u)^n(v)^m$$

$$+\beta z(A+I)\sum_{n,m=0}^{\infty}\sum_{s=0}^{n}\sum_{k=0}^{m}\frac{(-1)^{s+k}q^{\binom{k}{2}+\binom{m-k}{2}}}{(q;q)_s(q;q)_k}\frac{\left(q^{A+(s+k+2)I};q\right)_{n+m}}{(q;q)_{n-s}(q;q)_{m-k}}$$

$$\times\left[\left(q^{A+(s+k+1)I};q\right)_{(s+k)}\right]^{-1}\left(q^{-(A+(s+k+1)I)}\right)^{(s+k)}q^{\binom{s+k}{2}-(n+m)(s+k)}$$

$$\times\left(\beta zq^{-(A+(s+k+2)I)}\right)^{n+m-(s+k)}(\alpha x)^s(\alpha y)^k(u)^n(v)^m, \qquad (2.24)$$

Equating the coefficients of $(u)^n(v)^m$, we get the relation (2.16).
Similarly, differentiating (2.1) ,with respect to v, we get relation (2.17).

Theorem 2.4

The q-analogue modified Laguerre matrix polynomials of three variables $L_{n,m}^{(A,\alpha,\beta)}(x,y,z;q)$ satisfy the following relation:

$$\frac{\partial}{\partial z}L_{n,m}^{(A,\alpha,\beta)}(x,y,z;q)$$

$$=-\alpha\beta x\sum_{s=0}^{n-2}\sum_{k=0}^{m}\sum_{t=0}^{n-s-2}\sum_{p=0}^{m-k}\frac{q^{\binom{k}{2}+\binom{m-k-p}{2}+\binom{p}{2}+t+\binom{(s+t+k+p)}{2}-((n-2)+m)((s+t+k+p))}}{(q;q)_s(q;q)_k(q;q)_t(q;q)_p}$$

$$\times\frac{(q^{A+2I};q)_{t+p}\left(-q^{-(A+I)}\right)^{t+p}\left[\left(q^{-(A+((n-2)+m+s+k)I};q\right)_{(s+t+k+p)}\right]^{-1}}{(q;q)_{n-s-t-2}(q;q)_{m-k-p}}$$

$$\times\left(q^{A+(s+k+1)I};q\right)_{(n-2)+m}\left(\beta zq^{-(A+(s+k+1)I)}\right)^{(n-2)+m}\left(\frac{q\alpha x}{\beta z}\right)^s\left(\frac{q\alpha y}{\beta z}\right)^k$$

$$-\alpha\beta(x+y)\sum_{s=0}^{n-1}\sum_{k=0}^{m-1}\sum_{t=0}^{n-s-1}\sum_{p=0}^{m-k-1}\frac{q^{\binom{k}{2}+\binom{m-k-p-1}{2}+\binom{p}{2}+t+\binom{(s+t+k+p+1)}{2}-(n+m-2)((s+t+k+p+1))}}{(q;q)_s(q;q)_k(q;q)_t(q;q)_p}$$

$$\times \frac{(q^{A+2I};q)_{t+p}\left(-q^{-(A+I)}\right)^{t+p}\left[\left(q^{-(A+(n+m+s+k-2)I};q\right)_{(s+t+k+p+1)}\right]^{-1}}{(q;q)_{n-s-t-1}(q;q)_{m-k-p-1}}$$

$$\times\left(q^{A+(s+k+1)I};q\right)_{n+m-2}\left(\beta zq^{-(A+(s+k+1)I)}\right)^{(n-1)+(m-1)}\left(\frac{q\alpha x}{\beta z}\right)^{s}\left(\frac{q\alpha y}{\beta z}\right)^{k}$$

$$-\alpha\beta y\sum_{s=0}^{n}\sum_{k=0}^{m-2}\sum_{t=0}^{n-s}\sum_{p=0}^{m-k-2}\frac{q^{\binom{k}{2}+\binom{m-k-p-2}{2}+\binom{p}{2}+t+\binom{(s+t+k+p)}{2}-(n+(m-2))((s+t+k+p))}}{(q;q)_s(q;q)_k(q;q)_t(q;q)_p}$$

$$\times\frac{(q^{A+2I};q)_{t+p}\left(-q^{-(A+I)}\right)^{t+p}\left[\left(q^{-(A+(n+(m-2)+s+k)I};q\right)_{(s+t+k+p)}\right]^{-1}}{(q;q)_{n-s-t}(q;q)_{m-k-p-2}}$$

$$\times\left(q^{A+(s+k+1)I};q\right)_{n+(m-2)}\left(\beta zq^{-(A+(s+k+1)I)}\right)^{n+(m-2)}\left(\frac{q\alpha x}{\beta z}\right)^{s}\left(\frac{q\alpha y}{\beta z}\right)^{k}$$

$$+\beta(A+I)\sum_{s=0}^{n-1}\sum_{k=0}^{m}\frac{q^{\binom{k}{2}+\binom{m-k}{2}+\binom{s+k}{2}-((n-1)+m)(s+k)}}{(q;q)_s(q;q)_k}\frac{\left(q^{A+(s+k+2)I};q\right)_{(n-1)+m}}{(q;q)_{n-s-1}(q;q)_{m-k}}$$

$$\times\left[\left(q^{A+(s+k+1)I};q\right)_{(s+k)}\right]^{-1}\left(\beta zq^{-(A+(s+k+2)I)}\right)^{(n-1)+m}\left(\frac{q\alpha x}{\beta z}\right)^{s}\left(\frac{q\alpha y}{\beta z}\right)^{k}$$

$$+\beta(A+I)\sum_{s=0}^{n}\sum_{k=0}^{m-1}\frac{q^{\binom{k}{2}+\binom{m-k-1}{2}+\binom{s+k}{2}-(n+(m-1))(s+k)}}{(q;q)_s(q;q)_k}\frac{\left(q^{A+(s+k+2)I};q\right)_{n+(m-1)}}{(q;q)_{n-s}(q;q)_{m-k-1}}$$

$$\times\left[\left(q^{A+(s+k+1)I};q\right)_{(s+k)}\right]^{-1}\left(\beta zq^{-(A+(s+k+2)I)}\right)^{n+(m-1)}\left(\frac{q\alpha x}{\beta z}\right)^{s}\left(\frac{q\alpha y}{\beta z}\right)^{k}. \quad (2.25)$$

Proof. Differentiating (2.1), with respect to z, we get

$$\sum_{n,m=0}^{\infty}\frac{\partial}{\partial z}L_{n,m}^{(A,\alpha,\beta)}(x,y,z;q)\,u^n v^m$$

$$=[1-\beta qz(u+v)]_q^{-(A+I)}\left(\frac{-\alpha\beta(xu+yv)(u+v)}{(1-\beta z(u+v))(1-\beta qz(u+v))}\right)exp_q\left(\frac{-\alpha(xu+yv)}{1-\beta z(u+v)}\right)$$

$$+\beta(A+I)(u+v)[1-\beta z(u+v)]_q^{-(A+2I)}exp_q\left(\frac{-\alpha(xu+yv)}{1-\beta z(u+v)}\right), \quad (2.26)$$

applying relation (1.19) in (2.26), we get

$$\sum_{n,m=0}^{\infty}\frac{\partial}{\partial z}L_{n,m}^{(A,\alpha,\beta)}(x,y,z;q)\,u^n\,v^m=\left(-\alpha\beta(xu^2+xuv+yuv+yv^2)\right)$$

$$\times[1-\beta qz(u+v)]_q^{-(A+2I)}\sum_{s=0}^{\infty}\frac{(-1)^s\alpha^s[xu+yv]_q^s}{(q;q)_s}[1-\beta z(u+v)]_q^{-(A+(s+1)I)}$$

$$+\beta(A+I)(u+v)\sum_{s=0}^{\infty}\frac{(-1)^s\alpha^s[xu+yv]_q^s}{(q;q)_s}[1-\beta z(u+v)]_q^{-(A+(s+2)I)}, \quad (2.27)$$

Hence, by continuing the above steps of theorem (2.3), we get the required relation (2.25).

References

1. Aktas¸ R., Ekim, B. C.¸ Ahin, R. S.¸(2012). The matrix version for the multivariable Humbert polynomials, Miskolc Mathematical Notes 13 (2), 197-208.
2. Altın, A., Ekim, B. C.¸ (2012). Generating matrix functions for Chebyshev matrix polynomials of the second kind, Hacettepe Journal of Mathematics and Statistics 41 (1), 25-32.
3. Bin-Saad, Maged G., Anter, A. Al-Sayaad, (2015). Study of two variable Laguerre polynomials Via symbolic operational images. Asian J. Math. Comput. Res.2(1), 42-50.

4. [4] Defez, E., J´odar, L., (1998). Some applications of the Hermite matrix polynomials series expansions, Journal of Computational and Applied Mathematics 99, 105–117.
5. Dunford, N., Schwartz, J. ,(1963). Linear Operators, (1st edition), Interscience, New York.
6. Ekim, B. C¸ Altın, A., Aktas¸ R., (2011). Some relations satisfied by orthogonal matrix polynomials, Hacettepe Journal of Mathematics and Statistics 40 (2), 241-253.
7. Ekim, B. C¸(2013). New kinds of matrix polynomials, Miskolc Mathematical Notes 14 (3), 817-826.
8. Gasper, G and Rahman, M, (2004), Basic Hypergeometric Series, 2nd edn. Cambridge University Press, Cambridge.
9. J´odar L., Sastre, J. , (1998). On the Laguerre matrix polynomials, Utilitas Mathematica 53, 37-48.
10. J´odar, L., Cortes, J.C. , (1998). On the hypergeometric matrix function, Journal of Computational and Applied Mathematics 99, 205-217.
11. J´odar, L., Company, R., Navarro, E., (1994). Laguerre matrix polynomials and systems of second order di_erential equations, Applied Numerical Mathematics 15, 53-63.
12. J´odar L. , Sastre, J. , (2000). The growth of Laguerre matrix polynomials on bounded intervals, Applied Mathematics Letters 13, 21–26.
13. J´odar L. , Sastre, J. , (2004). Asymptotic expressions of normalized Laguerre matrix polynomials on bounded intervals, Utilitas Mathematica 65, 3–31.
14. Kargin, L., Kurt, V., (2014). Modified Laguerre matrix polynomials, Published by Faculty of Sciences and Mathematics, University of Niˇs, Serbia, Filomat 28:10 2069–2076.
15. Moak, D. S. , (1981). The q-analogue of the Laguerre polynomials. J. Math. Anal. Appl., 81, 20–47.
16. Mohsen, F.B.F., Alsarahi, F.S.N., (2018). q-Analogue modified Laguerre and generalized Laguerre polynomials of two variables, Hadhramout University Journal of Natural & Applied Sciences, Volume 15, Issue 2, 207-219.
17. Purohit, S.D. and Raina, R.K., (2010). Generalized q-Taylor's series and applications, General Mathematics Vol.18, No. 3, 19-28.
18. Sastre, J., Jodar, L. , (2006). On the asymptotics of Laguerre matrix polynomials, Utilitas Mathematica 70, 71-98.
19. Sastre, J., Defez, E. , (2006). On the asymptotics of Laguerre matrix polynomials for large x and n, Applied Mathematics Letters 19 (8), 721-727.
20. Sastre, J., J´odar, L., (2006). On Laguerre matrix polynomials series, Utilitas Mathematica 71, 109-130.
21. Srivastava, H.M. and Choi, J., (2012), Zeta and q-Zeta functions and associated series and integrals, Elsevier publications, USA and UK.

مصفوفة كثيرات حدود لأجير المعدلة الأساسية ذات ثلاثة متغيرات

فضل صالح ناصر السرحي
قسم الرياضيات – كلية التربية يافع– جامعة عدن - اليمن
DOI: https://doi.org/10.47372/uajnas.2021.n1.a12

الملخص

في هذه البحث قدمنا مصفوفة كثيرات حدود لأجير المعدلة الأساسية ذات ثلاثة متغيرات كمسلسلات نهائية. وبعض خصائص هذه المصفوفة أيضًا أوجدت.

الكلمات المفتاحية: مصفوفة كثيرات حدود لأجير المعدلة الأساسية ذات ثلاثة متغيرات، الدوال المولدة، العلاقات التكرارية.