

Research Article

New Exact Solutions for Generalized of Combined with Negative Calogero-Bogoyavlenskii Schiff and Generalized Yu–Toda–Sassa–Fukuyama Equations

M.S. Al-Amry^{1*} and E. F. Al-Abdali¹

¹ Department of Mathematics Faculty of Education Aden, Aden University

<https://doi.org/10.47372/uajnas.2024.n1.a04>

ARTICLE INFO	Abstract
<p>Received: 12 May 2024 Accepted: 23 Jun 2024</p> <p>Keywords: Calogero-Bogoyavlenskii Schiff (CBS) equation, negative-order Calogero-Bogoyavlenskii Schiff (nCBS) equation, Yu–Toda–Sassa–Fukuyama (YTSF) equation and the extended hyperbolic function method</p>	<p>In this paper, we present generalized model of combined Calogero-Bogoyavlenskii Schiff and negative-order Calogero-Bogoyavlenskii Schiff G(CBS-nCBS) equation and generalized Yu–Toda–Sassa–Fukuyama g(YTSF) equation. We apply the extended hyperbolic function method, to solve generalized models. Exact travelling wave solutions are obtained and expressed in terms of hyperbolic functions, trigonometric functions, rational functions solutions of these equations from the method with the aid of the computer program Maple.</p>

1. Introduction

The investigation of the travelling wave solutions for nonlinear partial differential equations plays an important role in the study of nonlinear physical phenomena. Nonlinear wave phenomena appear in various scientific and engineering fields, such as fluid mechanics, plasma physics, optical fibres, biology, solid state physics, chemical kinematics, chemical physics and geochemistry. Nonlinear wave phenomena of dispersion, dissipation, diffusion, reaction and convection are very important in nonlinear wave equations. New exact solutions may help to find new phenomena.

In order to get exact solutions directly, many powerful methods have been introduced such as the $(\frac{G'}{G})$ -expansion method [1,2], Hirota’s bilinear method [3], the tanh-coth method [4-6], the tan-cot method [7,8], the sine-cosine method [9], Bäcklund transformation method [10], the homogeneous balance [11], Darboux transformation [12], the Jacobi elliptic function expansion method [13,14], the mapping method [15-18]. In this paper, some new solutions

of g(CBS-nCBS) equation and g(YTSF) equation by using the extended hyperbolic function method [19].

2. The Extended Hyperbolic Function Method

Consider the general nonlinear partial differential equations (NLPDEs), say, in two variables,

$$P(u, u_x, u_y, u_z, u_t, u_{xx}, u_{xt}, \dots) = 0 \tag{1}$$

Eq. (1) can be solved by using the following steps:

Step 1

Use the wave variable $\xi = \lambda(x + y + z - \omega t)$, where λ is the wave number and ω is the wave speed to change the PDE (1) in to ODE

$$Q(u, u', u'', \dots) = 0, ' \equiv \frac{d}{d\xi} \tag{2}$$

In the above equation ' denotes to the differentiation with respect to ξ .

Step 2

We suppose that the solution of Eq. (2) has the form

$$u(x, y, z, t) = u(\xi) = a_0 + \sum_{i=1}^n a_i (f(\xi))^i, \tag{3}$$

* Correspondence to: M.S. Al-Amry, Department of Mathematics, University of Aden- Yemen
E-mail address: salem251251@gmail.com

where the coefficients $a_0, a_i, (i = 1, 2, \dots, n), \lambda$ and ω are constants to be determined, the parameter n will be found by balancing the highest – order nonlinear terms with the highest –order partial derivative term in the given equation, and the function $f(\xi)$ satisfies a nonlinear ordinary differential equation

$$\frac{df(\xi)}{d\xi} = a + bf^2(\xi), \tag{4}$$

where a and b are constants.

Step 3

Substituting Eq. (3) into Eq.(2) and using Eq. (4), collect the coefficients with the same order of $(f(\xi))^i, (i = 1, 2, \dots, n)$ and set the coefficients to zero, nonlinear algebraic equations are acquired. Solutions to the resulting algebraic system are derived by using the extended hyperbolic function method with the aid of the computer program Maple.

The ODE (4) has the following solutions

1. $f(\xi) = \operatorname{sng}(a) \sqrt{\frac{a}{b}} \tan(\sqrt{ab}\xi), \quad ab > 0,$
2. $f(\xi) = -\operatorname{sng}(a) \sqrt{\frac{a}{b}} \cot(\sqrt{ab}\xi), \quad ab > 0,$
3. $f(\xi) = \operatorname{sng}(a) \sqrt{-\frac{a}{b}} \tanh(\sqrt{-ab}\xi), \quad ab < 0,$
4. $f(\xi) = \operatorname{sng}(a) \sqrt{-\frac{a}{b}} \coth(\sqrt{-ab}\xi), \quad ab < 0,$
5. $f(\xi) = -\frac{1}{b\xi}, \quad a = 0, b > 0,$
6. $f(\xi) = a\xi, \quad a \in R, b = 0.$

The multiple exact special solutions of nonlinear partial differential equation (1) are obtained by making use of Eq. (3) and the solutions of ODE (4).

3. Applications

In this section, we determine the new exact traveling wave solutions of the nonlinear g(CBS - nCBS) and g(YTSF) equations by using the extended hyperbolic function method.

3.1. New Exact Solutions for G(CBS-nCBS) Equation

We consider generalized combined the (2+1)-dimensional Calogero-Bogoyavlenskii Schiff and negative-order Calogero-Bogoyavlenskii Schiff g(CBS-nCBS) equation [20,21] as the form

$$v_t + v_{xxy} + v_{xxt} + 4v^m(v_y + v_t) + 2mv_x \partial_x^{-1} v^{m-1}(v_y + v_t) = 0, \tag{5}$$

Assuming

$$u = \partial_x^{-1} mv^{m-1}(v_y + v_t)$$

implies $u_x = mv^{m-1}(v_y + v_t),$

and using the transformation $v(x, y, t) = v(\xi), \xi = \lambda(x + y - \omega t)$ in Eq. (5) we find

$$-\omega v'(\xi) + \lambda^2(1 - \omega)v'''(\xi) + 4(1 - \omega)v^m(\xi)v'(\xi) + 2u(\xi)v'(\xi) = 0, \tag{6, a}$$

$$u'(\xi) = m(1 - \omega)v^{m-1}v'(\xi). \tag{6, b}$$

Integrating the equation in the system (6,b) and neglecting the constants of integration, we find

$$u(\xi) = (1 - \omega)(v(\xi))^m. \tag{7}$$

Substituting Eq. (7) into the equation of the system (6,a) and integrating once the resulting equation, we find

$$-\omega v(\xi) + \lambda^2(1 - \omega)v''(\xi) + \frac{6(1-\omega)}{m+1}(v(\xi))^{m+1} = 0. \tag{8}$$

Eq. (8) is nonlinear ordinary differential equation.

Balancing the highest order of the nonlinear term $(v)^{m+1}$ with the highest order derivative v'' gives $nm + n = n + 2$ that gives $n = \frac{2}{m}$. To obtain closed form solutions, n should be a positive integer. To achieve this goal we use the transformation

$$v(\xi) = w^{\frac{1}{m}}(\xi), \quad 2 < m \in Z, \tag{9}$$

that will carry out Eq. (8) to the ODE

$$-\omega m^2(1 + m)w^2(\xi) + 6m^2(1 - \omega)w^3(\xi) + \lambda^2(1 - \omega)(1 - m^2)(w'(\xi))^2 + m\lambda^2(1 - \omega)(1 + m)w(\xi)w''(\xi) = 0. \tag{10}$$

Balancing the highest order of the nonlinear term w^3 with the highest order derivative ww'' gives $3n = 2n + 2$ that gives $n = 2$. Now, we apply the extended hyperbolic function method, to solve our g(CBS-nCBS) equation. Consequently, we get the original solutions for our g(CBS-nCBS) equation as the follows:

From Eq. (3), the solution of Eq. (10) has the form

$$w(x, y, t) = w(\xi) = a_0 + a_1 f(\xi) + a_2 f^2(\xi), \tag{11}$$

where a_0, a_1 and a_2 are constants.

By substituting Eq. (11) in Eq. (10) and using Eq. (4), the left hand side is converted into polynomials in $(f(\xi))^i, 0 \leq i \leq 6$. Setting each coefficient of these resulted polynomials to zero, we obtain a set of algebraic equations for a_0, a_1, a_2, ω and λ . Solving the system of algebraic equations with the aid of the computer program Maple, we obtain

$$a_0 = \frac{-ab\lambda^2(m+1)(m+2)}{3m^2}, a_1 = 0, a_2 = \frac{-\lambda^2 b^2(m+1)(m+2)}{3m^2},$$

$$\omega = \frac{4ab\lambda^2}{4ab\lambda^2 - m^2}, \lambda = \lambda.$$

Using Eq. (11) with the values of $[a_0, a_1, a_2, \omega]$, and the solutions of Eq. (4), we obtain

Case 1. For $ab > 0,$

$$v_1(x, y, t) = \left(\frac{-ab\lambda^2(m+1)(m+2)}{3m^2} \operatorname{sec}^2 \left(\lambda \sqrt{ab} \left(x + y - \frac{4ab\lambda^2 t}{4ab\lambda^2 - m^2} \right) \right) \right)^{\frac{1}{m}},$$

$$u_1(x, y, t) = \frac{ab\lambda^2(m+1)(m+2)}{3(4ab\lambda^2 - m^2)} \operatorname{sec}^2 \left(\lambda \sqrt{ab} \left(x + y - \frac{4ab\lambda^2 t}{4ab\lambda^2 - m^2} \right) \right),$$

$$v_2(x, y, t) = \left(\frac{-ab\lambda^2(m+1)(m+2)}{3m^2} \operatorname{csc}^2 \left(\lambda \sqrt{ab} \left(x + y - \frac{4ab\lambda^2 t}{4ab\lambda^2 - m^2} \right) \right) \right)^{\frac{1}{m}},$$

$$u_2(x, y, t) = \frac{ab\lambda^2(m+1)(m+2)}{3(4ab\lambda^2 - m^2)} \operatorname{csc}^2 \left(\lambda \sqrt{ab} \left(x + y - \frac{4ab\lambda^2 t}{4ab\lambda^2 - m^2} \right) \right).$$

Case 2. For $ab < 0,$

$$v_3(x, y, t) = \left(\frac{-ab\lambda^2(m+1)(m+2)}{3m^2} \operatorname{sech}^2 \left(\lambda \sqrt{-ab} \left(x + y - \frac{4ab\lambda^2 t}{4ab\lambda^2 - m^2} \right) \right) \right)^{\frac{1}{m}},$$

$$u_3(x, y, t) = \frac{ab\lambda^2(m+1)(m+2)}{3(4ab\lambda^2 - m^2)} \operatorname{sech}^2 \left(\lambda\sqrt{-ab} \left(x + y - \frac{4ab\lambda^2 t}{4ab\lambda^2 - m^2} \right) \right),$$

$$v_4(x, y, t) = \left(\frac{ab\lambda^2(m+1)(m+2)}{3m^2} \operatorname{csch}^2 \left(\lambda\sqrt{-ab} \left(x + y - \frac{4ab\lambda^2 t}{4ab\lambda^2 - m^2} \right) \right) \right)^{\frac{1}{m}},$$

$$u_4(x, y, t) = \frac{-ab\lambda^2(m+1)(m+2)}{3(4ab\lambda^2 - m^2)} \operatorname{csch}^2 \left(\lambda\sqrt{-ab} \left(x + y - \frac{4ab\lambda^2 t}{4ab\lambda^2 - m^2} \right) \right).$$

Case 3. For $a = 0, b > 0$,

$$v_5(x, y, t) = \left(\frac{-(m+1)(m+2)}{3m^2(x+y)^2} \right)^{\frac{1}{m}},$$

$$u_5(x, y, t) = \frac{-(m+1)(m+2)}{3m^2(x+y)^2}$$

3.2 New Exact Solutions for G(YTSF) Equation

We consider generalized the (3+1)- dimensional Yu–Toda–Sassa–Fukuyama g(YTSF) equation [22]

$$-4u_t + u_{xxz} + 4u^m u_z + 2mu_x \partial_x^{-1} u^{m-1} u_z + 3 \partial_x^{-1} u_{yy} = 0, u =$$

$$u(x, y, z, t), \quad (12)$$

assuming $v = \partial_x^{-1} mu^{m-1}u_z$ and $Q = \partial_x^{-1} u_{yy}$

implies $v_x = mu^{m-1}u_z$ and $Q_x = u_{yy}$,

and using the transformation $v(x, y, z, t) = v(\xi), Q(x, y, z, t) =$

$Q(\xi), \xi = \lambda(x + y + z - \omega t)$ in Eq. (12) we find

$$4\omega u'(\xi) + \lambda^2 u'''(\xi) + 4u^m(\xi)u'(\xi) + 2v(\xi)u'(\xi) +$$

$$3Q(\xi) = 0, \quad (13, a)$$

$$v'(\xi) = mu^{m-1}u'(\xi), \quad (13, b)$$

$$Q'(\xi) = u''(\xi). \quad (13, c)$$

Integrating the equations in the system (13,b) (13,c) and neglecting the constants of integration, we find

$$v(\xi) = (u(\xi))^m, \quad (14, a)$$

$$Q(\xi) = u'(\xi). \quad (14, b)$$

Substituting Eq. (14,a) and Eq. (14,b) into the equation of the system (13,a) and integrating the resulting equation, we find

$$(4\omega + 3)u(\xi) + \lambda^2 u''(\xi) + \frac{6}{m+1} (u(\xi))^{m+1} = 0. \quad (15)$$

Eq. (15) is nonlinear ordinary differential equation.

Balancing the highest order of the nonlinear term $(u)^{m+1}$ with the highest order derivative u'' gives $nm + n = n + 2$ that gives $n = \frac{2}{m}$. To obtain closed form solutions, n should be a positive integer. To achieve this goal we use the transformation

$$u(\xi) = w^{\frac{1}{m}}(\xi), \quad 2 < m \in Z, \quad (16)$$

that will carry out Eq. (15) to the ODE

$$(4\omega + 3)m^2(1+m)w^2(\xi) + 6m^2w^3(\xi) + \lambda^2(1-m^2)(w'(\xi))^2 + m\lambda^2(1+m)w(\xi)w''(\xi) = 0 \quad (17)$$

Balancing the highest order of the nonlinear term w^3 with the highest order derivative ww'' gives $3n = 2n + 2$ that gives $n = 2$. Now, we apply the extended hyperbolic

function method, to solve our g(YTSF) equation. Consequently, we get the original solutions for our g(YTSF) equation as the follows:

From Eq. (3), the solution of Eq. (17) has the form

$$w(x, y, z, t) = w(\xi) = a_0 + a_1 f(\xi) + a_2 f^2(\xi), \quad (18)$$

where a_0, a_1 and a_2 are constants.

By substituting Eq. (18) in Eq. (17) and using Eq. (4), the left hand side is converted into polynomials in $(f(\xi))^i, 0 \leq i \leq 6$. Setting each coefficient of these resulted polynomials to zero, we obtain a set of algebraic equations for a_0, a_1, a_2, ω and λ . Solving the system of algebraic equations with the aid of the computer program Maple, we obtain

$$a_0 = \frac{-ab\lambda^2(m+1)(m+2)}{3m^2}, a_1 = 0, a_2 = \frac{-\lambda^2 b^2(m+1)(m+2)}{3m^2},$$

$$\omega = \frac{4ab\lambda^2 - 3m^2}{4m^2}, \lambda = \lambda.$$

Using Eq. (18) with the values of $[a_0, a_1, a_2, \omega]$, and the solutions of Eq. (4) we get:

Case 1. For $ab > 0$,

$$u_1(x, y, z, t) =$$

$$\left(\frac{-ab\lambda^2(m+1)(m+2)}{3m^2} \operatorname{sech}^2 \left(\lambda\sqrt{ab} \left(x + y + z - \frac{4ab\lambda^2 - 3m^2}{4m^2} t \right) \right) \right)^{\frac{1}{m}},$$

$$v_1(x, y, z, t) = \frac{-ab\lambda^2(m+1)(m+2)}{3m^2} \operatorname{sech}^2 \left(\lambda\sqrt{ab} \left(x + y + z - \frac{4ab\lambda^2 - 3m^2}{4m^2} t \right) \right),$$

$$Q_1(x, y, z, t) = \frac{2\lambda\sqrt{ab}}{m} \tan \left(\lambda\sqrt{ab} \left(x + y + z - \frac{4ab\lambda^2 - 3m^2}{4m^2} t \right) \right)$$

$$\left(\frac{-ab\lambda^2(m+1)(m+2)}{3m^2} \operatorname{sech}^2 \left(\lambda\sqrt{ab} \left(x + y + z - \frac{4ab\lambda^2 - 3m^2}{4m^2} t \right) \right) \right)^{\frac{1}{m}}.$$

$$u_2(x, y, z, t) =$$

$$\left(\frac{-ab\lambda^2(m+1)(m+2)}{3m^2} \operatorname{csc}^2 \left(\lambda\sqrt{ab} \left(x + y + z - \frac{4ab\lambda^2 - 3m^2}{4m^2} t \right) \right) \right)^{\frac{1}{m}},$$

$$v_2(x, y, z, t) = \frac{-ab\lambda^2(m+1)(m+2)}{3m^2} \operatorname{csc}^2 \left(\lambda\sqrt{ab} \left(x + y + z - \frac{4ab\lambda^2 - 3m^2}{4m^2} t \right) \right),$$

$$Q_2(x, y, z, t) = \frac{-2\lambda\sqrt{ab}}{m} \cot \left(\lambda\sqrt{ab} \left(x + y + z - \frac{4ab\lambda^2 - 3m^2}{4m^2} t \right) \right)$$

$$\left(\frac{-ab\lambda^2(m+1)(m+2)}{3m^2} \operatorname{csc}^2 \left(\lambda\sqrt{ab} \left(x + y + z - \frac{4ab\lambda^2 - 3m^2}{4m^2} t \right) \right) \right)^{\frac{1}{m}}.$$

Case 2. For $ab < 0$,

$$u_3(x, y, z, t) =$$

$$\left(\frac{-ab\lambda^2(m+1)(m+2)}{3m^2} \operatorname{sech}^2 \left(\lambda\sqrt{-ab} \left(x + y + z - \frac{4ab\lambda^2 - 3m^2}{4m^2} t \right) \right) \right)^{\frac{1}{m}},$$

$$v_3(x, y, z, t) =$$

$$\frac{-ab\lambda^2(m+1)(m+2)}{3m^2} \operatorname{sech}^2 \left(\lambda\sqrt{-ab} \left(x + y + z - \frac{4ab\lambda^2 - 3m^2}{4m^2} t \right) \right),$$

$$Q_3(x, y, z, t) = \frac{-2\lambda\sqrt{-ab}}{m} \tanh\left(\lambda\sqrt{-ab}\left(x + y + z - \frac{4ab\lambda^2 - 3m^2}{4m^2}t\right)\right)$$

$$\left(\frac{-ab\lambda^2(m+1)(m+2)}{3m^2} \operatorname{sech}^2\left(\lambda\sqrt{-ab}\left(x + y + z - \frac{4ab\lambda^2 - 3m^2}{4m^2}t\right)\right)\right)^{\frac{1}{m}}$$

$$u_4(x, y, z, t) = \left(\frac{ab\lambda^2(m+1)(m+2)}{3m^2} \operatorname{csch}^2\left(\lambda\sqrt{-ab}\left(x + y + z - \frac{4ab\lambda^2 - 3m^2}{4m^2}t\right)\right)\right)^{\frac{1}{m}}$$

$$v_4(x, y, z, t) = \frac{ab\lambda^2(m+1)(m+2)}{3m^2} \operatorname{csch}^2\left(\lambda\sqrt{-ab}\left(x + y + z - \frac{4ab\lambda^2 - 3m^2}{4m^2}t\right)\right),$$

$$Q_4(x, y, z, t) = \frac{-2\lambda\sqrt{-ab}}{m} \coth\left(\lambda\sqrt{-ab}\left(x + y + z - \frac{4ab\lambda^2 - 3m^2}{4m^2}t\right)\right)$$

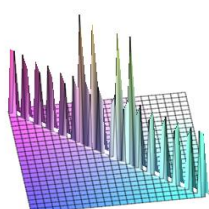
$$\left(\frac{ab\lambda^2(m+1)(m+2)}{3m^2} \operatorname{csch}^2\left(\lambda\sqrt{-ab}\left(x + y + z - \frac{4ab\lambda^2 - 3m^2}{4m^2}t\right)\right)\right)^{\frac{1}{m}}$$

Case 3. For $a = 0, b > 0$,

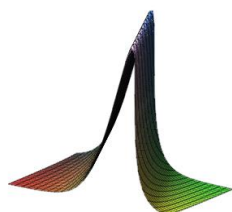
$$u_5(x, y, z, t) = \left(\frac{-(m+1)(m+2)}{3m^2\left(x + y + z + \frac{3}{4}t\right)^2}\right)^{\frac{1}{m}}$$

$$v_5(x, y, z, t) = \frac{-(m+1)(m+2)}{3m^2\left(x + y + z + \frac{3}{4}t\right)^2},$$

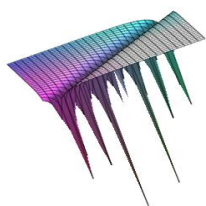
$$Q_5(x, y, z, t) = \frac{-2}{m\left(x + y + z + \frac{3}{4}t\right)} \left(\frac{-(m+1)(m+2)}{3m^2\left(x + y + z + \frac{3}{4}t\right)^2}\right)^{\frac{1}{m}}$$



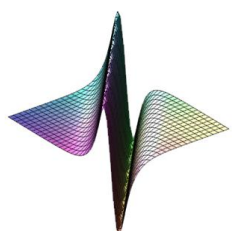
$u_5(x, y, z, t), m=3$



$v_5(x, y, z, t), m=3, a=-1, b=1, \lambda=1$



$u_4(x, y, z, t), m=3, a=-1, b=1, \lambda=1$



$Q_3(x, y, z, t), \lambda=1, a=-1, b=1, m=3$

Conclusion

In this paper, the extended hyperbolic function method has been successfully implemented to find new traveling waves solutions for generalized proposed equations namely, a combined Calogero-Bogoyavlenskii Schiff and negative-order Calogero-Bogoyavlenskii Schiff g(CBS-nCBS) equation and generalized Yu–Toda–Sassa–Fukuyama g(YTTSF) equation. The results show that this method is a powerful Mathematical tool for obtaining new exact solutions for our equations. It is also a promising method to solve other nonlinear partial differential equations.

References

- Hossain, Akm Kazi Sazzad; Akbar, M. Ali. Closed form solutions of two nonlinear equation via the enhanced (G'/G)-expansion method. *Cogent Mathematics*, 2017, 4.1: 1355958.
- Mohanty, Sanjaya Kr; Kravchenko, Oleg V.; Dev, Apul N. Exact traveling wave solutions of the Schamel Burgers' equation by using generalized-improved and generalized G'/G expansion methods. *Results in Physics*, 2022, 33: 105124.
- ZHANG, Da-Jun; JI, Jie; ZHAO, Song-Lin. Soliton scattering with amplitude changes of a negative order AKNS equation. *Physica D: Nonlinear Phenomena*, 2009, 238.23-24: 2361-2367.
- Al-Askar, Farah M., et al. The Impact of the Wiener process on the analytical solutions of the stochastic (2+ 1)-dimensional breaking soliton equation by using tanh-coth method. *Mathematics*, 2022, 10.5: 817.
- Ananna, Samsun Nahar, et al. Solitary wave structures of a family of 3D fractional WBBM equation via the tanh-coth approach. *Partial Differential Equations in Applied Mathematics*, 2022, 5: 100237.
- Lin, Lin, et al. Exact solutions of Gardner equations through tanh-coth method. *Applied Mathematics*, 2016, 7.18: 2374-2381.
- Islam, Md Monirul, et al. On the Exact Soliton Solutions of Some Nonlinear PDE by Tan-Cot Method. *GUB Journal of Science and Engineering (GUBJSE)*, 2018, 5.1.
- Özkan, Yeşim Sağlam; Yaşar, Emrullah. On the exact solutions of nonlinear evolution equations by the improved tan ($\varphi/2$)-expansion method. *Pramana*, 2020, 94.1: 37.
- Wazwaz, A.-M. A sine-cosine method for handling nonlinear wave equations. *Mathematical and Computer modelling*, 2004, 40.5-6: 499-508.
- Raza, Nauman, et al. Complexiton and resonant multi-solitons of a (4+1) - dimensional Boiti–Leon–Manna–Pempinelli equation. *Optical and Quantum Electronics*, 2022, 54: 1-16.
- Yan, Zhenya; Zhang, Hongqing. New explicit solitary wave solutions and periodic wave solutions for Whitham–Broer–Kaup equation in shallow water. *Physics Letters A*, 2001, 285.5-6: 355-362.
- Chen, Junchao; Ma, Zhengyi; Hu, Yahong. Nonlocal symmetry, Darboux transformation and soliton–cnoidal wave interaction solution for the shallow water wave equation. *Journal of Mathematical Analysis and Applications*, 2018, 460.2: 987-1003.
- Ma, Wen-Xiu; Huang, Tingwen; Zhang, Yi. A multiple expansion function method for nonlinear differential equations and its application. *Physica Scripta*, 2010, 82.6: 065003.
- Tarla, Sibel, et al. New optical solitons based on the perturbed Chen–Lee–Liu model through Jacobi elliptic function method. *Optical and Quantum Electronics*, 2022, 54.2: 131.
- Peng, Y.Z. , Exact solutions for some nonlinear partial differential equations, *Physics Letters A*, (2003), Aug 11;314(5-6):401-8.

16. Maher, A.; El-Hawary, H. M.; Al-Amry, M. S. New exact solutions for new model nonlinear partial differential equation. *Journal of Applied Mathematics*, 2013, 2013.
17. Maher, A.; El-Hawary, H. M.; Al-Amry, M. S. The Mapping Method For Solving A New Model Of Nonlinear Partial Differential Equation. *International Journal of Pure and Applied Mathematics*, 2014, 95.2: 181-208.
18. Samir, Islam, et al. Solitary wave solutions and other solutions for Gilson–Pickering equation by using the modified extended mapping method. *Results in Physics*, 2022, 36: 105427.
19. Shang, Yadong; Huang, Yong; Yuan, Wenjun. The extended hyperbolic functions method and new exact solutions to the Zakharov equations. *Applied Mathematics and Computation*, 2008, 200.1: 110-122.
20. Wazwaz, Abdul-Majid. A new integrable equation constructed via combining the recursion operator of the Calogero–Bogoyavlenskii–Schiff (CBS) equations and its inverse operator. *Appl. Math. Inf. Sci.*, 2017, 11.5: 1241-1246.
21. Wazwaz, Abdul-Majid. Negative-order forms for the Calogero–Bogoyavlenskii–Schiff equation and the modified Calogero–Bogoyavlenskii–Schiff equation. *Proceedings of the Romanian Academy. Series A*, 2017, 18.4: 337-344.
22. Wazwaz, Abdul-Majid. Multiple-soliton solutions for the Calogero–Bogoyavlenskii–Schiff, Jimbo–Miwa and YTSE equations. *Applied Mathematics and Computation*, 2008, 203.2: 592-597.



مجلة جامعة عدن للعلوم الطبيعية والتطبيقية

Journal homepage: <https://uajnas.adenuniv.com>



بحث علمي

الحلول الدقيقة الجديدة لمعادتي G(YTSF) و G(CBS-nCBS)

محمد سالم أحمد العمري و إيمان فضل عبدالله العبدلي
- كلية التربية عدن - جامعة عدن قسم الرياضيات

<https://doi.org/10.47372/uajnas.2024.n1.a04>

مفاتيح البحث	الملخص
<p>التسليم : 12 مايو 2024 القبول : 23 يونيو 2024</p> <p>كلمات مفتاحية : معادلة كالوجرو- بوجوفلنسكي شيف العادية (CBS)، معادلة كالوجرو- بوجوفلنسكي شيف ذات الترتيب السلبي (nCBS)، معادلة يو- تودا- ساسا- فوكوياما (YTSF) و طريقة الدالة الزائدية الموسعة</p>	<p>في هذه البحث قدمنا نموذجاً معممًا لمعادلة كالوجرو- بوجوفلنسكي شيف العادية (CBS)، معادلة كالوجرو- بوجوفلنسكي شيف ذات الترتيب السلبي (nCBS) ومعادلة يو- تودا- ساسا- فوكوياما (YTSF). ثم قمنا بتطبيق طريقة الدالة الزائدية الموسعة لحل النماذج المعممة. حيث تم الحصول على حلول الموجات المتنقلة الدقيقة والتعبير عنها من حيث الوظائف الزائدية والوظائف المثلثية وحلول الوظائف المنطقية لهذه المعادلات من الطريقة بمساعدة برنامج مابل. أظهرت النتائج أن هذه الطريقة هي أداة رياضية قوية للحصول على حلول دقيقة لمعادلاتنا. وهي أيضاً طريقة واحدة لحل المعادلات الجزئية غير الخطية الأخرى.</p>