On a generalized $\beta K$ – Birecurrent .......... Fahmi Yaseen Abdo Qasem and Wafa'a Hadi Ali Hadi

On a generalized $\beta K$ – Birecurrent Finsler space
Fahmi Yaseen Abdo Qasem and Wafa'a Hadi Ali Hadi
Email: fahmi.yaseen@yahoo.com Email: wf_hadi@yahoo.com
Dep. of Math., Faculty of Edu. – Aden, Univ. of Aden, Khormaksar, Aden, Yemen
DOI: https://doi.org/10.47372/uajnas.2018.n1.a14

Abstract

In the present paper, we introduced a Finsler space whose Cartan’s fourth curvature tensor $K^i_{jkh}$ satisfies the condition

$$B_nB_mB^i_{jkh} = a_{mn}K^i_{jkh} + b_{mn}(\delta^i_kg_{jh} - \delta^i_hg_{jk}) - 2y^r\mu_nB_r(\delta^i_kC^m_{jhn} - \delta^i_hC^m_{jkn}),$$

where $B_nB_m$ are Berwald's covariant differential operator of the second order with respect to $x^m$ and $x^n$, successively, $B_r$ is Berwald's covariant differential operator of the first order with respect to $x^r$. $a_{mn}$ and $b_{mn}$ are non-zero covariant tensors. Field of second order called "recurrence tensors" and $\mu_n$ is non-zero covariant vector field, such space is called a generalized $\beta K$ – birecurrent space.

The aim of this paper is to prove that the curvature tensor $H^i_{jkh}$ satisfies the generalized birecurrence property. We proved that Ricci tensors $H_{jk}, K_{jk}$, the curvature vector $H_k$, and the curvature scalar $H$ of such space are non-vanishing and under certain conditions, a generalized $\beta K$ – birecurrent space becomes Landsberg space. Also, some conditions have been pointed out which reduce a generalized $\beta K$ – birecurrent space $F_n(n > 2)$ into Finsler space of curvature scalar.

Keywords: Finsler space, Generalized $\beta K$ – birecurrent space, Ricci tensor, Landsberg space, Finsler space of curvature scalar.

1-Introduction

Ruse [7] considered a three dimensional Riemannian space having the recurrent of curvature tensor and he called such space a Riemannian space of recurrent curvature. This idea was extended to n-dimensional Riemannian and non-Riemannian space by Walker [8], Wong [9], Wong and Yano [10] and others. Dikshit [1] introduced a Finsler space whose Berwald curvature tensor $H^i_{jkh}$ satisfies the recurrence property in the sense of Berwald. Qasem and Saleem [5] discussed general Finsler space for the $hv$ – curvature tensor $U^i_{jkh}$ satisfies the birecurrence property with respect to Berwald's connection parameter $G^i_{jk}$ and they called it UBR- Finsler space. Pandey, Saxena and Goswami [3] introduced and discussed a Finsler space whose Berwald curvature tensor $H^i_{jkh}$ satisfying the generalized recurrence property in the sense of Berwald, they called such space generalized H-recurrent Finsler space. Qasem and Hadi [4] introduced and studied generalized $\beta R$ – birecurrent space.

Let us consider an n-dimensional Finsler space $F_n$ equipped with a metric function $F(x, y)$ and satisfying the restricted condition of Finslerian metric [6].

The vectors $y_i$, $y^i$, the metric tensor $g_{ij}$, its associative metric tensor $g^{ij}$ and the tensors $C_{ijk}$ and $G^i_{jkh}$ are satisfying the following relations:

(1.1) a) $y_i y^i = F^2$, b) $g_{ij} = \delta_i y_j = \delta_j y_i$, c) $B_k y^i = 0$.

d) $C_{ijk} y^i = C_{kij} y^i = C_{jki} y^i = 0$, e) $B_k g_{ij} = -2C_{ijkh} y^h = -2y^h B_k C_{ijk}$, f) $G_{jk} y^j = G_{jkh} y^j = G_{kjh} y^j = 0$ and g) $g_{ij} y^j = y_i$.

The unit vector $\{l\}$ and its associative vector $\l = \frac{y_i}{F}$ are defined by

(1.2a) $\l = \frac{y_i}{F}$ and (b) $\l = g_{ij} \l = \delta_i F = \frac{y_i}{F}$.

On a generalized $\beta K$ – Birecurrent 

The processes of Berwald’s covariant differentiation and the partial differentiation commute according to

\begin{equation}
(\partial k \mathcal{B}_h - \mathcal{B}_k \partial h) T^i_j = T^i_j \mathcal{G}_{kh} - T^i_j \mathcal{G}_{kh}. \tag{1.3}
\end{equation}

Berwald curvature tensor \( H^i_{kh} \) satisfies the relations

\begin{equation}
H^i_{kh} y^i = H^i_{kh}, \tag{1.4}
\end{equation}

and

\begin{equation}
H^i_{kjh} = \partial_j H^i_{kh}. \tag{1.5}
\end{equation}

The \( h(v)\)-torsion tensor \( H^i_{kh} \) satisfies

\begin{equation}
H^i_{kh} y^k = H^i_h, \tag{1.6}
\end{equation}

\begin{equation}
K^i_{kjh} y^j = H^i_{kh}, \tag{1.7}
\end{equation}

\begin{equation}
H^i_{jk} = H^i_{jki}, \tag{1.8}
\end{equation}

\begin{equation}
H^i_k = H^i_{ki}, \tag{1.9}
\end{equation}

and

\begin{equation}
H = \frac{1}{n-1} H^i_i. \tag{1.10}
\end{equation}

where \( H^i_h, K^i_{kjh}, H^i_k \) and \( H \) are called the deviation tensor of Berwald curvature tensor, Cartan’s third curvature tensor, \( H \)-Ricci tensor, curvature vector and curvature scalar, respectively.

Since contraction of the indices doesn’t affect the homogeneity in \( y^i \), hence the tensors \( H^i_{rk} \), \( H^i_k \) and \( H \) are also homogeneous of degree zero, one and two in \( y^i \), respectively. The above tensors are also connected by

\begin{equation}
H^i_{jk} y^j = H^i_k, \tag{1.11}
\end{equation}

\begin{equation}
H^i_{jk} = \partial_j H^i_k, \tag{1.12}
\end{equation}

and

\begin{equation}
H^i_k y^k = (n-1)H. \tag{1.13}
\end{equation}

The tensors \( H^i_h \) and \( H^i_{kh} \) satisfy the following:

\begin{equation}
H^i_{kh} = \partial_k H^i_h. \tag{1.14}
\end{equation}

The necessary and sufficient condition for a Finsler space \( F_n(n > 2) \) to be a Finsler space of scalar curvature is given by

\begin{equation}
H^i_h = F^2R(\delta^i_h - \delta^i_i). \tag{1.15}
\end{equation}

A Finsler space \( F_n \) is said to be Landsberg space if it satisfies

\begin{equation}
y^r G^i_{rjk} = 0. \tag{1.16}
\end{equation}

\( K - \) Ricci tensor \( K^i_{jk} \) is given by

\begin{equation}
K^i_{jki} = K^i_{jk}. \tag{1.17}
\end{equation}

2. Generalized $\beta K$ – Birecurrent Space

A Finsler space whose Cartan’s fourth curvature tensor \( K^i_{jkh} \) satisfies the condition

\begin{equation}
\mathcal{B}_n K^i_{jkh} = \lambda_n K^i_{jkh} + \mu_n (\delta^i_k g_{jh} - \delta^i_j g_{hk} ), K^i_{jkh} \neq 0, \tag{2.1}
\end{equation}

called a generalized \( K \)- recurrent space, where \( \lambda_n \) and \( \mu_n \) are non-zero covariant vectors field and called the recurrence vectors field.

Taking the covariant derivative for the condition (2.1) with respect to \( x^m \) in the sense of Berwaldand using (1.1e), we get

\begin{equation}
\mathcal{B}_m \mathcal{B}_n K^i_{jkh} = \mathcal{B}_m \lambda_n + \lambda_n \lambda_m) K^i_{jkh} + ( \lambda_n \mu_m + \mathcal{B}_m \mu_n)(\delta^i_k g_{jh} - \delta^i_j g_{hk}) - 2y^r \mu_n \mathcal{B}_r (\delta^i_k C_{jhm} - \delta^i_h C_{jkm}), \tag{2.2}
\end{equation}

which can be written as

\begin{equation}
\mathcal{B}_m \mathcal{B}_n K^i_{jkh} = a_{mn} K^i_{jkh} + b_{mn} (\delta^i_k g_{jh} - \delta^i_j g_{hk}) - 2y^r \mu_n \mathcal{B}_r (\delta^i_k C_{jhm} - \delta^i_h C_{jkm}), \tag{2.3}
\end{equation}

where \( a_{mn} = \mathcal{B}_m \lambda_n + \lambda_n \lambda_m \) and \( b_{mn} = \lambda_n \mu_m + \mathcal{B}_m \mu_n \) are non-zero covariant tensors field of second order.
On a generalized $\beta K$ – Birecurrent ………Fahmi Yaseen Abdo Qasem and Wafa’a Hadi Ali Hadi

Definition 2.1. AFinsler space $F_n$ whose Cartan’s fourth curvature tensor $K_{kh}^{jk}$ satisfies the condition (2.3) will be called generalized $\beta K$ - birecurrent space, we shall denote it briefly by $G\beta K – BRF_n$.

Transvecting the condition (2.3) by $y^j$, using (1.1c), (1.7), (1.1g) and (1.1d), we get

$$B_mB_n H_k^{ij} = a_m H_k^{ij} + b_m n \delta_k Y_n - \delta_l Y_k.$$  

Further, transvecting (2.4) by $y^k$, using (1.1c), (1.6) and (1.1a), we get

$$B_mB_n H^k = a_m H^k + b_m n \delta^i Y_n - \delta^l Y_k.$$  

Thus, we conclude

Theorem 2.1. In $\beta K – BRF_n$, Berwald’s covariant derivative of second order for the $h(v)$ – torsion tensor $H_k^{ij}$ and the deviation tensor $H^k_{ij}$ given by the conditions (2.4) and (2.5), respectively.

Contracting the indices $i$ and $h$ in the conditions (2.3), (2.4) and (2.5), separately, using (1.17), (1.9) and (1.10), we get

$$B_mB_n K_{jk} = a_m K_{jk} + (1 - n) b_m g_{jk} - 2 (1 - n) v^i \mu_n B_r C_{jkm}.$$  

Using commutation formula exhibited by (1.7), (1.8) and (1.9), we get

$$B_mB_n H_k = a_m H_k + (1 - n) b_m g_{jk}.$$  

and

$$B_mB_n H = a_m H - b_m F^2.$$  

The conditions (2.6), (2.7) and (2.8), show that $K - Ricci$ tensor $K_{jk}$, the curvature vector $H_k$ and the curvature scalar $H$ can’t vanish, because the vanishing of any one of them would imply $b_{mn} = 0$ and $\mu_{mn} = 0$, a contradiction.

Thus, we conclude

Theorem 2.2. In $\beta K – BRF_n$, $K - Ricci$ tensor $K_{jk}$, the curvature vector $H_k$ and the curvature scalar $H$ are non-vanishing.

Differentiating (2.7) partially with respect to $y^j$ and using (1.1b), we get

$$\partial_j (B_mB_n H_k) = (\partial_j a_m) H_k + a_m (\partial_j H_k) + (1 - n) \partial_j b_m g_{jk}.$$  

Using commutation formula exhibited by (1.3) for $(B_n H_k)$ in (2.9) and using (1.12), we get

$$B_mB_n H_k = (B_r H_k) G_{fmn} - (B_n H_r) G_{jmk} = (\partial_j a_m) H_k + a_m H_{jk} + (1 - n) \partial_j b_m g_{jk}.$$  

Again, applying the commutation formula exhibited by (1.3) for $(H_k)$ in (2.10), we get

$$B_mB_n H_{jk} = (B_r H_{jk}) G_{fkn} - (B_n H_r) G_{jmk} = (\partial_j a_m) H_k + a_m H_{jk} + (1 - n) \partial_j b_m g_{jk}.$$  

This shows that

$$B_mB_n H_{jk} = a_m H_{jk} + (1 - n) b_m g_{jk}$$  

if and only if

$$B_mB_n H_{jk} = - (B_r H_{jk}) G_{fkn} - (B_n H_r) G_{jmk} = (\partial_j a_m) H_k + a_m H_{jk} + (1 - n) \partial_j b_m g_{jk}.$$  

Thus, we conclude

Theorem 2.3. In $G\beta K – BRF_n$, $H - Ricci$ tensor $H_{jk}$ is non-vanishing if and only if (2.13) holds good.

Transvecting (2.11) by $y^k$, using (1.1c), (1.1f), (1.11), (1.13), (1.1g) and (1.1a), we get

$$B_mB_n H_j - (1 - n) (B_r H) G_{jmn} = (1 - n) (\partial_j a_m) H + a_m H_{j} + (n - 1) \partial_j b_m g_{ij}.$$  

Using the condition (2.7) in (2.14), we get

$$B_mB_n H^r = -(\partial_j a_m) H + (\partial_j b_m) F^2.$$  

Suppose $(B_r H) G_{jmn} = 0$, in view of (2.15), we get

$$- (\partial_j a_m) H + (\partial_j b_m) F^2 = 0.$$  

which can be written as

$$\partial_j b_m = \frac{(\partial_j a_m) H}{F^2}.$$  

On a generalized $\beta K$ – Birecurrent ……..Fahmi Yaseen Abdo Qasem and Wafa’a Hadi Ali Hadi

If the covariant tensor field $a_{mn}$ is independent of the directional argument $y^i$, the equation (2.17) shows that the covariant tensor field $b_{mn}$ is independent of the directional argument $y^i$. Conversely, if the covariant tensor field $b_{mn}$ is independent of the directional argument $y^i$, we get $H(\hat{\delta}_ja_{mn}) = 0$.

In view of theorem 2.2, the condition $H(\hat{\delta}_ja_{mn}) = 0$ implies $\hat{\delta}_ja_{mn} = 0$, i.e. the covariant tensor field $a_{mn}$ is also independent of $y^i$. This leads to

**Theorem 2.4.** In $\beta K - BRF_n$, the covariant tensor field $b_{mn}$ is independent of the directional argument and only if the covariant tensor field $a_{mn}$ is independent of the directional argument provided $(B_rH)G^r_{jmn} = 0$.

Suppose the covariant tensor field $a_{mn}$ is not independent of $y^i$, in view of (2.11), (2.12) and (2.17), we get

$$\tag{2.18} -B_m\left((H_rG^r_{jkn}) - (B_rH_k)G^j_{mn} - (B_nH_r)G^r_{jmk}\right) = \hat{\delta}_ja_{mn} (H_k - \frac{(n-1)}{f^2}Hy_k)$$

Transvecting (2.18) by $y^m$, using (1.1c) and (1.1) , we get

$$\tag{2.19} -B_m((H_rG^r_{jkn}))(y^m) = (\hat{\delta}_ja_{m})(y^m)(H_k - \frac{(n-1)}{f^2}Hy_k),$$

which implies

$$\tag{2.20} -B_m((H_rG^r_{jkn}))(y^m) = (\hat{\delta}_ja_{m} - a_{jm})(H_k - \frac{(n-1)}{f^2}Hy_k),$$

where $a_{mn}y^m = a_n$.

Suppose $B_m((H_rG^r_{jkn}))(y^m) = 0$, the equation (2.20) has at least one of the following conditions

$$\tag{2.21} \begin{align*}
a) & \quad a_{jn} = \hat{\delta}_ja_{n}, & b) & \quad H_k = \frac{(n-1)}{f^2}Hy_k.
\end{align*}$$

Thus, we conclude

**Theorem 2.5.** In $G\beta K - BRF_n$, which the covariant tensor field $a_{mn}$ is not independent of the directional argument at least one of the conditions (2.21a) and (2.21b) holds.

Suppose the condition (2.21b) holds, then (2.18) implies

$$\tag{2.22} -B_m\left((n-1)H(y_rG^r_{jkn}) - (B_r(n-1)H(y_k)G^j_{mn} - (B_n(n-1)H(y_r)G^r_{jmk}\right) = 0.$$}

Transvecting (2.22) by $y^m$, using (1.1c) and (1.1f), we get

$$\tag{2.23} B_m((H)\gamma_rG^r_{jkn})y^m + H(B_mG^r_{jkn})(\gamma_r) = 0.$$}

If $H(B_mG^r_{jkn})(\gamma_r) = 0$, then the equation (2.23) implies

$$\tag{2.24} y_rG^r_{jkn} = 0,$$

since $B_m((H)y^m) = 0$. Therefore, the space is Landsberg space.

Thus, we conclude

**Theorem 2.6.** An $G\beta K - BRF_n$ is Landsberg space if the condition (2.21b) holds provided $H(B_mG^r_{jkn})(\gamma_r)y^m = 0$.

If the covariant tensor field $a_{jn} \neq \hat{\delta}_ja_{n}$, in view of theorem 2.5, the condition (2.21b) holds. In view of this fact, we may rewrite theorem 2.6 in the following

**Theorem 2.7.** An $G\beta K - BRF_n$ is necessarily Landsberg space provided $a_{jn} \neq \hat{\delta}_ja_{n}$ and $H(B_mG^r_{jkn})(\gamma_r)y^m = 0$.

Differentiating the condition (2.4) partially with respect to $y^i$, using (1.5) and (1.1b), we get

$$\tag{2.25} \hat{\delta}_j(B_mB_nH_{kh}) = (\hat{\delta}_jc_{mn})H^i_{kh} + c_{mn}H^j_{kh} + 2(\hat{\delta}_jd_{mn})y^m \delta^i_ky_h - \delta^i_hy_k$$

$$+ d_{mn}(\delta^i_kg_{jh} - \delta^i_hg_{jk}).$$

Using commutation formula exhibited by (1.3) for $(B_nH^i_{kh})$ in (2.25), we get

$$\tag{2.26} B_mB_n(\hat{\delta}_jB_nH_{kh}) - (B_nH^i_{kh})G^r_{jmn} + (B_nH^i_{kh})(G^r_{jmr} - (B_nH^i_{rk})G^r_{jmk}$$

$$\quad - (B_nH^i_{hr})G^r_{jmk} = (\hat{\delta}_ja_{mn})H^i_{kh} + a_{mn}H^j_{kh} + (\hat{\delta}_jb_{mn})(\delta^i_ky_h - \delta^i_hy_k)$$

On a generalized $\beta K$ – Birecurrent ……..Fahmi Yaseen Abdo Qasem and Wafa’a Hadi Ali Hadi

Again applying the commutation formula exhibited by (1.3) for $(H^l_{jk})$ in (2.26) and using (1.5), we get

\[ B_m(B_nH^r_{jkh} + H^r_{jkh}G^l_{jnr} - H^r_{jrk}G^r_{jnh} - H^r_{hr}G^l_{jnk}) - (B_rH^l_{kh})G^l_{jmn} \]
\[ + (B_nH^l_{kh})G^r_{jmn} - (B_nH^l_{rk})G^l_{jmh} - (B_nH^r_{hr})G^r_{jnm} = (\delta_j a_{mn})H^l_{kh} \]
\[ + a_{mn}H^l_{jkh} + (\delta_j b_{mn})(\delta^l_k y_{hn} - \delta^l_k y_{hn}) + b_{mn}(\delta^l_k g_{jn} - \delta^l_k g_{jk}) . \]

This shows that

\[ B_m(B_nH^l_{jkh}) = a_{mn}H^l_{jkh} + b_{mn}(\delta^l_k g_{jn} - \delta^l_k g_{jk}) \]

if and only if

\[ (B_mH^r_{kh})G^l_{jnr} + H^r_{jkh}(B_nG^l_{jnr}) - (B_mH^l_{rk})G^r_{jnh} - H^l_{rk}(B_nG^r_{jnh}) \]
\[ - (B_mH^r_{hr})G^r_{jnk} - H^r_{hr}(B_nG^r_{jnk}) - (B_rH^l_{kh})G^l_{jmn} + (B_nH^r_{rk})G^r_{jmr} \]
\[ - (B_nH^r_{rk})G^r_{jmn} - (B_nH^r_{hr})G^l_{jmhl} = (\delta_j a_{mn})H^l_{kh} + (\delta_j b_{mn})(\delta^l_k y_{hn} - \delta^l_k y_{hn}) . \]

Thus, we conclude

**Theorem 2.8.** In $\beta K - BRF_n$, Berwald's covariant derivative of second order for the curvature tensor $H^l_{jkh}$ is given by the condition (2.28) if and only if (2.29) holds good.

Transvecting (2.29) by $y^k$, using (1.1c), (1.1f), (1.1a) and (1.6), we get

\[ (B_mH^l_{kh})G^l_{jnr} + H^r_{jkh}(B_nG^l_{jnr}) - (B_mH^l_{rk})G^r_{jnh} \]
\[ - (B_mH^r_{hr})G^r_{jnk} - H^r_{hr}(B_nG^r_{jnk}) - (B_rH^l_{kh})G^l_{jmn} + (B_nH^r_{rk})G^r_{jmr} \]
\[ - (B_nH^r_{rk})G^r_{jmn} - (B_nH^r_{hr})G^l_{jmhl} = (\delta_j a_{mn})H^l_{kh} + (\delta_j b_{mn})(\delta^l_k y_{hn} - \delta^l_k y_{hn}) . \]

In view of (2.17) and (2.30), we get

\[ (B_mH^l_{kh})G^l_{jnr} + H^r_{jkh}(B_nG^l_{jnr}) - (B_mH^l_{rk})G^r_{jnh} \]
\[ - (B_mH^r_{hr})G^r_{jnk} - H^r_{hr}(B_nG^r_{jnk}) - (B_rH^l_{kh})G^l_{jmn} + (B_nH^r_{rk})G^r_{jmr} \]
\[ - (B_nH^r_{rk})G^r_{jmn} - (B_nH^r_{hr})G^l_{jmhl} = (\delta_j a_{mn})[H^l_{kh} - H(\delta^l_k \{ \delta^l_k - [ l \{ l \} ] ) . \]

If

\[ (B_mH^r_{kh})G^l_{jnr} + H^r_{jkh}(B_nG^l_{jnr}) - (B_mH^l_{rk})G^r_{jnh} \]
\[ - (B_mH^r_{hr})G^r_{jnk} - H^r_{hr}(B_nG^r_{jnk}) - (B_rH^l_{kh})G^l_{jmn} + (B_nH^r_{rk})G^r_{jmr} \]
\[ - (B_nH^r_{rk})G^r_{jmn} - (B_nH^r_{hr})G^l_{jmhl} = 0 . \]

We have at least one of the following conditions

\[ a) \delta_j a_{mn} = 0 , \quad b) H^l_{kh} = H(\delta^l_k - [ l \{ l \} ) . \]

Putting $H = F^2R$, $R \neq 0$, the condition (2.33b) becomes

\[ H^l_{kh} = F^2R(\delta^l_k - [ l \{ l \} ) . \]

Therefore the space is a Finsler space of curvature scalar.

Thus, we conclude

**Theorem 2.9.** An $G\beta K - BRF_n$, for $(n > 2)$ admitting $B_mH^l_{km}G^l_{jnr} + H^r_{m}(B_nG^l_{jnr}) - (B_mH^r_{kh})G^r_{jnh} - H^r_{k}(B_nG^r_{jnh}) - (B_rH^l_{kh})G^l_{jmh} + (B_nH^r_{rk})G^r_{jmr} = 0$ is a Finsler space of curvature scalar provided $R \neq 0$ and the covariant tensor filed $a_{mn}$ is not independent of the directional argument.
On a generalized $\beta K$ – Birecurrent ……..Fahmi Yaseen Abdo Qasem and Wafa’a Hadi Ali Hadi

References
فضاء فنسلر المعمم ثانائي المعاودة

فهمي ياسين عبده قاسم و وفاء هادي علي هادي
قسم الرياضيات، كلية التربية/عدن, جامعة عدن, خورمكسر، عدن, اليمن

Email: fahmi.yaseen@yahoo.com & Email: wf_hadi@yahoo.com
DOI: https://doi.org/10.47372/uajnas.2018.n1.a14

المتخصِّص

في هذه الورقة قدمنا فضاء فنسلر الذي فيه الموتر الرابع لكارتanian

\[ K_{jkh} \]

يحقق الخاصية الآتية:

\[ B_m B_n K_{jkh} = a_{mn} K_{jkh} + b_{mn} (\delta_k^l g_{jh} - \delta_h^l g_{jk}) - 2 y^r \mu_n B_r (\delta_l^j C_{jnm} - \delta_h^j C_{jkm}) , \]

حيث

\[ B_m, B_n \]

هي المُشتقة الثانية لبروالد المُتجمعة بالاتجاه لـ \( x^m, x^n \) على التعاقب،

\[ \mu_n, a_{mn}, b_{mn} \]

موترات حقل مُتجمعة غير صفري من الرتبة الثانية وتشمّي موترات حقل أحادية المعاودة و

\[ \beta K \]

حقهق الخاصية المعممة ثانوية المعاودة وأثبتنا أن

في هذه الورقة أثبتنا أن نموذج الديسسي \( H_{jkh} \)

\[ \text{مُجَّلَّح} \]

العلاقة المعممة ثانوية المعاودة و أثبتنا أن

موترات ريتشاري \( (H_{jk}, K_{jk}) \)

وثابت التقوس

\[ H_{k} \]

لا تنتهي في الفضاء المعمم ثانائي المعاودة- \( \beta K \)

أثبتنا أن فضاء فنسلر المعمم ثانائي المعاودة

\[ \beta K \]

هو فضاء لامبارج.

وأخيراً أوجدنا بعض المبرهنات والحالات التي تختزل فضاء فنسلر المعمم ثانائي المعاودة- \( \beta K \)

فضاء فنسلر (\( n > 2 \)) ثابت التقوس.

الكلمات مفتاحية: فضاء فنسلر، فضاء فنسلر المعمم ثانائي المعاودة- \( \beta K \)

موترات ريتشاري. فضاء لامبارج. فضاء فنسلر ثابت التقوس.