On a generalized βK −Birecurrent Finsler space

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Abstract

In the present paper, we introduceda Finsler space whose Cartan's fourth curvature tensor K^i_{jkh} satisfies the condition

$$\mathcal{B}_n\mathcal{B}_m K^i_{jkh} = a_{mn}K^i_{jkh} + b_{mn}\left(\delta^i_k g_{jh} - \delta^i_h g_{jk}\right) - 2y^r\mu_n\mathcal{B}_r(\delta^i_k C_{jhm} - \delta^i_h C_{jkm}),$$

where $\mathcal{B}_n\mathcal{B}_m$are Berwald's covariant differential operator of the second order with respect to $x^m$and x^n, successively, $\mathcal{B}_r$ is Berwald's covariant differential operator of the first order with respect to x^r, a_{mn} and $b_{mn}$are non-zero covariant tensors field of second order called *recurrence tensorsfield* and μ_n is non-zero covariant vector field , such space is called as a *generalized βK −birecurrent space*.

The aim of this paper is to prove that thecurvature tensor H^i_{jkh}satisfies the generalized birecurrence property .We proved that Ricci tensors H_{jk}, K_{jk}, the curvature vector H_k and the curvature scalar$H$of such space are non-vanishing andunder certain conditions, a generalized βK −birecurrent space becomes Landsberg space . Also, some conditions have been pointed out which reduce a generalized βK −birecurrent space $F_n(n > 2)$ into Finsler space of curvature scalar.

Keywords: Finsler space, Generalized βK −birecurrent space, Ricci tensor,Landsberg space,Finsler space of curvature scalar.

1-Introduction

Ruse[7] considered a three dimensional Riemannian space having the recurrent of curvature tensor and he called such space a *Riemannian space of recurrent curvature*. This idea was extended to n-dimensional Riemannian and non- Riemannian space by Walker [8], Wong [9] , Wong and. Yano [10] and others. Dikshit [1] introduced a Finsler space whose Berwald curvature tensor H^i_{jkh} satisfies the recurrence property in the sense of Berwald..Qasemand.Saleem [5] discussed general Finsler space for the hv −curvature tensor U^i_{jkh} satisfies the birecurrence property with respect to Berwald's connection parameter G^i_{jk} and they called it *UBR- Finsler space*.Pandey,Saxena and Goswami[3] introduced and discussed a Finsler space whose Berwald curvature tensor H^i_{jkh} satisfing the generalized recurrence property in the sense of Berwald, they called such space *generalized H-recurrent Finsler space* .Qasem and Hadi [4] introduced and studied generalized βR −birecurrent space .

Let us consider an n- dimensional Finsler space $F_n$equipped with a metric function $F(x, y)$ and satisfying the requestic condition of Finslerian metric [6] .

The vectors y_i , y^i , the matric tensor g_{jj} ,its associative metric tensor g^{ij} and the tensors C_{ijk} and $G^i_{jkh}$are satisfying the following relations:

(1.1) a) $y_i\, y^i = F^2$,b) $g_{ij} = \dot{\partial}_i y_j = \dot{\partial}_j y_i$, c) $\mathcal{B}_k\, y^i = 0$,

d) $C_{ijk}\, y^i = C_{kij}\, y^i = C_{jki}\, y^i = 0$,

e) $\mathcal{B}_k\, g_{ij} = -2C_{ijkh}y^h = -2y^h\mathcal{B}_h C_{ijk}$,

f) $G^i_{jkh}\, y^j = G^i_{hjk}\, y^j = G^i_{khj}\, y^j = 0$ and g) $g_{ij}\, y^j = y_i$.

The unit vector $l^i$and its associative vector l_i are defined by

(1.2)$a)$ $l^i = \frac{y^i}{F}$ and $b)$ $l_i = g_{ij}l^j = \dot{\partial}_i F = \frac{y_i}{F}$.

The processes of Berwald's covariant differentiation and the partial differentiation commute according to

(1.3) $(\dot\partial_k \mathcal{B}_h - \mathcal{B}_k \dot\partial_h)\, T^i_j = T^r_j G^i_{khr} - T^i_r G^r_{khj}$.

Berwald curvature tensor H^i_{jkh} satisfies the relations

(1.4) $H^i_{jkh}\, y^j = H^i_{kh}$

and

(1.5) $H^i_{jkh} = \dot\partial_j H^i_{kh}$.

The h(v)- torsion tensor H^i_{kh} satisfies

(1.6) $H^i_{kh}\, y^k = H^i_h$,

(1.7) $K^i_{jkh}\, y^j = H^i_{kh}$,

(1.8) $H_{jk} = H^i_{jki}$,

(1.9) $H_k = H^i_{ki}$,

and

(1.10) $H = \frac{1}{n-1} H^i_i$.

where H^i_h , K^i_{jkh} , H_{jk} , H_k and H are called *the deviation tensor of Berwald curvature tensor* , *Cartan'sthird curvature tensor*,*H-Ricci tensor*, *curvature vector*and *curvature scalar*,respectively. Since contraction of the indices doesn't affect the homogeneity in y^i, hence the tensors H_{rk} , $H_r$and H are also homogeneous of degree zero, one and two in y^i, respectively . The above tensors are also connected by

(1.11) $H_{jk}\, y^j = H_k$,

(1.12) $H_{jk} = \dot\partial_j H_k$

and

(1.13) $H_k\, y^k = (n-1)H$.

The tensors H^i_h and H^i_{kh} satisfy the following :

(1.14) $H^i_{kh} = \dot\partial_k H^i_h$

The necessary and sufficient condition for a Finsler space $F_n (n > 2)$ to be a Finsler space of scalar curvature is given by

(1.15) $H^i_h = F^2 R(\delta^i_h - l^i l_h)$.

A Finsler space F_n is said to be *Landsberg space* if it satisfies

(1.16) $y_r G^r_{ijk} = 0$.

$K -$ Ricci tensor K_{jk} is given by

(1.17) $K^i_{jki} = K_{jk}$.

2. Generalized βK −Birecurrent Space

A Finsler space whose Cartan's fourth curvature tensor K^i_{jkh} satisfies the condition

(2.1) $\mathcal{B}_n K^i_{jkh} = \lambda_n K^i_{jkh} + \mu_n(\delta^i_k g_{jh} - \delta^i_h g_{jk})$, $K^i_{jkh} \neq 0$,

called a *generalized K- recurrent space*, where λ_n and μ_n are non-zero covariant vectors field and called the *recurrence vectors field.*

Taking the covariant derivative for the condition (2.1) with respect to x^m in the sense of Berwaldand using (1.1e) ,we get

(2.2) $\mathcal{B}_m \mathcal{B}_n K^i_{jkh} = (\mathcal{B}_m \lambda_n + \lambda_n \lambda_m) K^i_{jkh} + (\lambda_n \mu_m + \mathcal{B}_m \mu_n)(\delta^i_k g_{jh} - \delta^i_h g_{jk})$
$-2y^r \mu_n \mathcal{B}_r (\delta^i_k C_{jhm} - \delta^i_h C_{jkm})$.

which can be written as

(2.3) $\mathcal{B}_m \mathcal{B}_n K^i_{jkh} = a_{mn} K^i_{jkh} + b_{mn}(\delta^i_k g_{jh} - \delta^i_h g_{jk}) - 2y^r \mu_n \mathcal{B}_r (\delta^i_k C_{jhm} - \delta^i_h C_{jkm})$,

where $a_{mn} = \mathcal{B}_m \lambda_n + \lambda_n \lambda_m$ and $b_{mn} = \lambda_n \mu_m + \mathcal{B}_m \mu_n$ are non-zero covariant tensors field of second order.

Definition2.1.AFinsler space F_n whose Cartan's fourth curvature tensor K^i_{jkh} satisfies the condition (2.3) will be called *generalized βK-birecurrentspace*,we shall denote it briefly by$G\beta K - BRF_n$.

Transvecting the condition (2.3) by y^j , using (1.1c) , (1.7), (1.1g) and (1.1d), we get

(2.4) $\mathcal{B}_m\mathcal{B}_n H^i_{kh} = a_{mn} H^i_{kh} + b_{mn}(\delta^i_k y_h - \delta^i_h y_k)$.

Further, transvecting (2.4) by y^k , using (1.1c) , (1.6) and (1.1a) , we get

(2.5) $\mathcal{B}_m\mathcal{B}_n H^i_h = a_{mn} H^i_h + b_{mn}(y^i y_h - \delta^i_h F^2)$.

Thus, we conclude

Theorem2.1. In $\beta K - BRF_n$, Berwald's covariant derivative of second order for the $h(v)$ −torsion tensor H^i_{kh} and the deviation tensor H^i_h given by the conditions (2.4) and (2.5) ,respectively.

Contracting the indices i and h in the conditions (2.3), (2.4) and (2.5) ,separately , using (1.17) ,(1.9) and (1.10) , we get

(2.6) $\mathcal{B}_m\mathcal{B}_n K_{jk} = a_{mn} K_{jk} + (1-n) b_{mn} g_{jk} - 2(1-n) y^r \mu_n \mathcal{B}_r C_{jkm}$,

(2.7) $\mathcal{B}_m\mathcal{B}_n H_k = a_{mn} H_k + (1-n) b_{mn} y_k$

and

(2.8) $\mathcal{B}_m\mathcal{B}_n H = a_{mn} H - b_{mn} F^2$.

The conditions (2.6), (2.7) and (2.8), show that K −Ricci tensor K_{jk}, the curvature vector H_k and the curvature scalar H can't vanish , because the vanishing of any one of them would imply$b_{mn} = 0$ and $\mu_{mn} = 0$, a contradiction .

Thus, we conclude

Theorem2.2. In $\beta K - BRF_n$, $K-$ Ricci tensor K_{jk}, the curvature vectorH_k and the curvature scalar H are non-vanishing .

Differentiating (2.7) partially with respect to y^j and using (1.1b), we get

(2.9) $\dot\partial_j(\mathcal{B}_m\mathcal{B}_n H_k) = (\dot\partial_j a_{mn}) H_k + a_{mn}(\dot\partial_j H_k) + (1-n)(\dot\partial_j b_{mn}) y_k + (1-\ \ n)\, b_{mn} g_{jk}$.

Using commutation formula exhibited by (1.3) for ($\mathcal{B}_n H_k$) in (2.9) and using (1.12), we get

(2.10) $\mathcal{B}_m \dot\partial_j(\mathcal{B}_n H_k) - (\mathcal{B}_r H_k) G^r_{jmn} - (\mathcal{B}_n H_r) G^r_{jmk} = (\dot\partial_j a_{mn}) H_k$
$+ a_{mn} H_{jk} + (1-n)(\dot\partial_j b_{mn}) y_k + (1-n) b_{mn} g_{jk}$.

Again, applying the commutation formula exhibited by (1.3) for (H_k) in (2.10), we get

(2.11) $\mathcal{B}_m\mathcal{B}_n H_{jk} - \mathcal{B}_m(H_r G^r_{knj}) - (\mathcal{B}_r H_k) G^r_{jmn} - (\mathcal{B}_n H_r) G^r_{jmk} = (\dot\partial_j a_{mn}) H_k + a_{mn} H_{jk} +$
$(1-n)(\dot\partial_j b_{mn}) y_k + (1-n) b_{mn} g_{jk}$.

This shows that

(2.12) $\mathcal{B}_m\mathcal{B}_n H_{jk} = a_{mn} H_{jk} + (1-n) b_{mn} g_{jk}$

if and only if

(2.13) $-\mathcal{B}_m(H_r G^r_{knj}) - (\mathcal{B}_r H_k) G^r_{jmn} - (\mathcal{B}_n H_r) G^r_{jmk} = (\dot\partial_j a_{mn}) H_k + (1-n)(\dot\partial_j b_{mn}) y_k$.

Thus, we conclude

Theorem2.3. In $G\beta K - BRF_n$, H −Ricci tensor H_{jk} is non-vanishing if and only if (2.13) holds good.

Transvecting (2.11) by y^k , using (1.1c) , (1.1f), (1.11), (1.13), (1.1g) and (1.1a), we get

(2.14)$\mathcal{B}_m\mathcal{B}_n H_j - (1-n)(\mathcal{B}_r H) G^r_{jmn} = (1-n)(\dot\partial_j a_{mn}) H + a_{mn} H_j + \quad (n-1)(\dot\partial_j b_{mn}) +$
$(n-1) b_{mn} y_j$.

Using the condition (2.7) in (2.14), we get

(2.15) $(\mathcal{B}_r H) G^r_{jmn} = -(\dot\partial_j a_{mn}) H + (\dot\partial_j b_{mn}) F^2$.

Suppose $(\mathcal{B}_r H) G^r_{jmn} = 0$, in view of (2.15) , we get

(2.16) $-(\dot\partial_j a_{mn}) H + (\dot\partial_j b_{mn}) F^2 = 0$

which can be written as

(2.17) $\dot\partial_j b_{mn} = \frac{(\dot\partial_j a_{mn}) H}{F^2}$.

If the covariant tensor field a_{mn} is independent of the directional argument y^i, the equation (2.17) shows that the covariant tensor field b_{mn} is independent of the directional argument y^i . Conversely , if the covariant tensor b_{mn} is independent of the directional argument y^i , we get $H(\dot{\partial}_j a_{mn}) = 0$.

In view of theorem2.2 , the condition $H(\dot{\partial}_j a_{mn}) = 0$ implies $\dot{\partial}_j a_{mn} = 0$,i.e. the covariant tensor field a_{mn} is also independent of y^i . This leads to

Theorem2.4. In $\beta K - BRF_n$ **,** the covariant tensor field b_{mn} is independent of the directional argumentif and only ifthe covariant tensor field a_{mn} is independent of the directional argumentprovided$(\mathcal{B}_r H) G^r_{jmn} = 0$.

Suppose the covariant tensor field a_{mn} is not independent of y^i, in view of (2.11) , (2.12) and (2.17) , we get

(2.18) $$-\mathcal{B}_m(H_r G^r_{knj}) - (\mathcal{B}_r H_k) G^r_{jmn} - (\mathcal{B}_n H_r) G^r_{jmk} = \dot{\partial}_j a_{mn}(H_k - \tfrac{(n-1)}{F^2} H y_k).$$

Transvecting (2.18) by y^m, using (1.1c) and (1.1f) , we get

(2.19) $$-\mathcal{B}_m(H_r G^r_{knj}) y^m = (\dot{\partial}_j a_{mn}) y^m (H_k - \tfrac{(n-1)}{F^2} H y_k),$$

which implies

(2.20) $$-\mathcal{B}_m(H_r G^r_{knj}) y^m = (\dot{\partial}_j a_n - a_{jn})(H_k - \tfrac{(n-1)}{F^2} H y_k),$$

where $a_{mn} y^m = a_n$

Suppose $\mathcal{B}_m(H_r G^r_{knj}) y^m = 0$, the equation (2.20) has at least one of the following conditions

(2.21) a) $a_{jn} = \dot{\partial}_j a_n$, b) $H_k = H_k - \frac{(n-1)}{F^2} H y_k$.

Thus, we conclude

Theorem2.5. In $G\beta K - BRF_n$, which the covariant tensor field a_{mn} is not independent of the directional argument at least one of the conditions(2.21a) and (2.21b) holds.

Suppose the condition (2.21b) holds, then (2.18) implies

(2.22) $$-\mathcal{B}_m\left(\tfrac{(n-1)H}{F^2} y_r G^r_{knj}\right) - \left(\mathcal{B}_r \tfrac{(n-1)H}{F^2} y_k\right) G^r_{jmn} - \left(\mathcal{B}_n \tfrac{(n-1)H}{F^2} y_r\right) G^r_{jmk} = 0.$$

Transvecting (2.22) by y^m, using (1.1c) and (1.1f), we get

(2.23) $$\mathcal{B}_m(H) y_r G^r_{knj} y^m + H(\mathcal{B}_m G^r_{knj}) y_r y^m = 0.$$

If $H(\mathcal{B}_m G^r_{knj}) y_r y^m = 0$, then the equation (2.23) implies

(2.24) $$y_r G^r_{knj} = 0 ,$$

since $\mathcal{B}_m(H) y^m \neq 0$. Therefore, the space is Landsberg space .

Thus , we conclude

Theorem2.6. An $G\beta K - BRF_n$ is Landsberg space if the condition (2.21b) holds provided $H(\mathcal{B}_m G^r_{knj}) y_r y^m = 0$.

If the covariant tensor field $a_{jn} \neq \dot{\partial}_j a_n$, in view of theorem2.5 , the condition (2.21b) holds. In view of this fact , we may rewrite theorem2.6 in the following

Theorem2.7. An $G\beta K - BRF_n$ is necessarily Landsberg space provided $a_{jn} \neq \dot{\partial}_j a_n$ and $H(\mathcal{B}_m G^r_{knj}) y_r y^m = 0$.

Differentiating the condition (2.4) partially with respect to y^j , using (1.5) and (1.1b), we get

(2.25) $$\dot{\partial}_j(\mathcal{B}_m \mathcal{B}_n H^i_{kh}) = (\dot{\partial}_j c_{mn}) H^i_{kh} + c_{mn} H^i_{jkh} + (\dot{\partial}_j d_{mn})(\delta^i_k y_h - \delta^i_h y_k) + d_{mn}(\delta^i_k g_{jh} - \delta^i_h g_{jk}) .$$

Using commutation formula exhibited by (1.3) for $(\mathcal{B}_n H^i_{kh})$ in (2.25) , we get

(2.26) $$\mathcal{B}_m(\dot{\partial}_j \mathcal{B}_n H^i_{kh}) - (\mathcal{B}_r H^i_{kh}) G^r_{jmn} + (\mathcal{B}_n H^r_{kh}) G^i_{jmr} - (\mathcal{B}_n H^i_{rk}) G^r_{jmh} - (\mathcal{B}_n H^i_{hr}) G^r_{jmk} = (\dot{\partial}_j a_{mn}) H^i_{kh} + a_{mn} H^i_{jkh} + (\dot{\partial}_j b_{mn})(\delta^i_k y_h - \delta^i_h y_k)$$

$+b_{mn}(\delta^i_k g_{jh} - \delta^i_h g_{jk})$.

Again applying the commutation formula exhibited by (1.3) for (H^i_{kh}) in (2.26) and using (1.5), we get

$$\mathcal{B}_m\big(\mathcal{B}_n H^i_{jkh} + H^r_{kh} G^i_{jnr} - H^i_{rk} G^r_{jnh} - H^i_{hr} G^r_{jnk}\big) - (\mathcal{B}_r H^i_{kh}) G^r_{jmn} + (\mathcal{B}_n H^r_{kh}) G^i_{jmr} - (\mathcal{B}_n H^i_{rk}) G^r_{jmh} - (\mathcal{B}_n H^i_{hr}) G^r_{jmk} = (\dot\partial_j a_{mn}) H^i_{kh} + a_{mn} H^i_{jkh} + (\dot\partial_j b_{mn})\big(\delta^i_k y_h - \delta^i_h y_k\big) + b_{mn}\big(\delta^i_k g_{jh} - \delta^i_h g_{jk}\big)$$

the above equation can be written as

(2.27) $$\mathcal{B}_m \mathcal{B}_n H^i_{jkh} + (\mathcal{B}_m H^r_{kh}) G^i_{jnr} + H^r_{kh}(\mathcal{B}_m G^i_{jnr}) - (\mathcal{B}_m H^i_{rk}) G^r_{jnh} - H^i_{rk}(\mathcal{B}_m G^r_{jnh}) - \big(\mathcal{B}_m H^i_{hr}\big) G^r_{jnk} - H^i_{hr}(\mathcal{B}_m G^r_{jnk}) - (\mathcal{B}_r H^i_{kh}) G^r_{jmn} + (\mathcal{B}_n H^r_{kh}) G^i_{jmr} - (\mathcal{B}_n H^i_{rk}) G^r_{jmh} - (\mathcal{B}_n H^i_{hr}) G^r_{jmk} = (\dot\partial_j a_{mn}) H^i_{kh} + a_{mn} H^i_{jkh} + (\dot\partial_j b_{mn})\big(\delta^i_k y_h - \delta^i_h y_k\big) + b_{mn}(\delta^i_k g_{jh} - \delta^i_h g_{jk})\ .$$

This shows that

(2.28) $$\mathcal{B}_m \mathcal{B}_n H^i_{jkh} = a_{mn} H^i_{jkh} + b_{mn}(\delta^i_k g_{jh} - \delta^i_h g_{jk})$$

if and only if

(2.29) $$(\mathcal{B}_m H^r_{kh}) G^i_{jnr} + H^r_{kh}(\mathcal{B}_m G^i_{jnr}) - (\mathcal{B}_m H^i_{rk}) G^r_{jnh} - H^i_{rk}(\mathcal{B}_m G^r_{jnh}) - \big(\mathcal{B}_m H^i_{hr}\big) G^r_{jnk} - H^i_{hr}(\mathcal{B}_m G^r_{jnk}) - (\mathcal{B}_r H^i_{kh}) G^r_{jmn} + (\mathcal{B}_n H^r_{kh}) G^i_{jmr} - (\mathcal{B}_n H^i_{rk}) G^r_{jmh} - (\mathcal{B}_n H^i_{hr}) G^r_{jmk} = (\dot\partial_j a_{mn}) H^i_{kh} + (\dot\partial_j b_{mn})\big(\delta^i_k y_h - \delta^i_h y_k\big).$$

Thus, we conclude

Theorem2.8. In $\beta K - BRF_n$, Berwald's covariant derivative of second order for the curvature tensor $H^i_{jkh}$is given by the condition (2.28) if and only if (2. 29) holds good.

Transvecting (2.29) by y^k , using (1.1c) , (1.1f), (1.1a) and (1.6), we get

(2.30) $$(\mathcal{B}_m H^r_h) G^i_{jnr} + H^r_h(\mathcal{B}_m G^i_{jnr}) - (\mathcal{B}_m H^i_r) G^r_{jnh} - H^i_r(\mathcal{B}_m G^r_{jnh}) - (\mathcal{B}_r H^i_h) G^r_{jmn} + (\mathcal{B}_n H^r_h) G^i_{jmr} - (\mathcal{B}_n H^i_r) G^r_{jmh} = \big(\dot\partial_j a_{mn}\big) H^i_h - (\dot\partial_j b_{mn})\big(\delta^i_h F^2 - y^i y_h\big)\ .$$

In view of (2.17) and (2.30), we get

(2.31) $$(\mathcal{B}_m H^r_h) G^i_{jnr} + H^r_h(\mathcal{B}_m G^i_{jnr}) - (\mathcal{B}_m H^i_r) G^r_{jnh} - H^i_r(\mathcal{B}_m G^r_{jnh}) - (\mathcal{B}_r H^i_h) G^r_{jmn} + (\mathcal{B}_n H^r_h) G^i_{jmr} - (\mathcal{B}_n H^i_r) G^r_{jmh} = \big(\dot\partial_j a_{\ell m}\big)[H^i_h - H\big(\delta^i_h - \mathfrak{l}^i\, \mathfrak{l}_h\big)].$$

If

(2.32) $$(\mathcal{B}_m H^r_h) G^i_{jnr} + H^r_h(\mathcal{B}_m G^i_{jnr}) - (\mathcal{B}_m H^i_r) G^r_{jnh} - H^i_r(\mathcal{B}_m G^r_{jnh}) - (\mathcal{B}_r H^i_h) G^r_{jmn} + (\mathcal{B}_n H^r_h) G^i_{jmr} - (\mathcal{B}_n H^i_r) G^r_{jmh} = 0\ .$$

We have at least one of the following conditions

(2.33) a) $\dot\partial_j a_{mn} = 0$, b) $H^i_h = H(\delta^i_h - \mathfrak{l}^i\, \mathfrak{l}_h)$.

Putting $H = F^2 R\ , R \neq 0$, the condition (2.33b) becomes

(2.34) $$H^i_h = F^2 R(\delta^i_h - \mathfrak{l}^i\, \mathfrak{l}_h).$$

Therefore the space is a Finsler space of curvature scalar .

Thus , we conclude

Theorem2.9. An $G\beta K - BRF_n$, for $(n > 2)$ admitting$(\mathcal{B}_m H^r_h) G^i_{jnr} + H^r_h(\mathcal{B}_m G^i_{jnr}) - (\mathcal{B}_m H^i_r) G^r_{jnh} - H^i_r(\mathcal{B}_m G^r_{jnh}) - (\mathcal{B}_r H^i_h) G^r_{jmn} + (\mathcal{B}_n H^r_h) G^i_{jmr} - (\mathcal{B}_n H^i_r) G^r_{jmh} = 0$ is a Finsler space of curvature scalar provided $R \neq 0$ and the covariant tensor filed a_{mn} is not independent of the directional argument.

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فضاء فنسلر المُعمم ثُنائي المُعاودة -βK

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المُلخص

في هذه الورقة قدمنا فضاء فنسلر الذي فيه الموتر الرابع لكارتان K^i_{jkh} يُحقق الخاصية الاتية:
$\mathcal{B}_m\mathcal{B}_n K^i_{jkh} = a_{mn}K^i_{jkh} + b_{mn}(\delta^i_k g_{jh} - \delta^i_h g_{jk}) - 2y^r\mu_n\mathcal{B}_r(\delta^i_k C_{jhm} - \delta^i_h C_{jkm})$,
حيث $\mathcal{B}_m\mathcal{B}_n$ هي المُشتقة الثانية لبروالد المُتحدة الاختلاف بالنسبة الى x^n، x^m على التعاقب، b_{mn}، a_{mn} موترات حقل متحدة الاختلاف غير صفرية من الرتبة الثانية وتُسمى موترات حقل أُحادية المُعاودة و μ_n مُتجه حقل مُتحد الاختلاف غير صفري وسمينا هذا الفضاء بفضاء فنسلر المُعمم ثُنائي المُعاودة $\beta K-$.
في هذه الورقة أثبتنا أنَّ موتر بروالد التقوسي H^i_{jkh} يُحقق الخاصية المُعممة ثُنائية المُعاودة وأثبتنا أنَّ موترات ريتشي (H_{jk}, K_{jk})، المُتجه التقوسي H_k وثابت التقوس H لا تنتهي في الفضاء المُعمم ثُنائي المُعاودة− βK .
أثبتنا أنَّ فضاء فنسلر المُعمم ثُنائي المُعاودة $\beta K-$ هو فضاء لامبارج.
وأخيراً أوجدنا بعض المُبرهنات والحالات التي تختزل فضاء فنسلر المُعمم ثُنائي المُعاودة-βK إلى فضاء فنسلر $F_n(n > 2)$ ثابت التقوس.

الكلمات مفتاحية: فضاء فنسلر, فضاء فنسلر المُعمم ثُنائي المُعاودة $\beta K-$, موترات ريتشي, فضاء لامبارج, فضاء فنسلر ثابت التقوس.