Integrals formulas involving confluent hypergeometric Functions of three variables $\Phi_2^{(3)}$ and $\Psi_2^{(3)}$

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Abstract

The aim of this paper is to establish two general integral formulas involving confluent hypergeometric functions of three variables $\Phi_2^{(3)}$ and $\Psi_2^{(3)}$ with the help of two extension formulas for Lauricella's functions of three variables $F_A^{(3)}$ and $F_D^{(3)}$ due to Atash [1] and Atash and Bellehaj [2]. Some applications of our main results are also presented.

Keywords: Integral formulas, Hypergeometric functions, Dixon's theorem, Kummer's theorem

1. Introduction

The Lauricella's functions of three variables $F_A^{(3)}$ and $F_D^{(3)}$ are defined and represented as follows [7]:

$$F_A^{(3)}(a,b_1,b_2,b_3;c_1,c_2,c_3;x_1,x_2,x_3) = \sum_{m_1,m_2,m_3=0}^{\infty} \frac{(a)_{m_1+m_2+m_3}(b_1)_{m_1}(b_2)_{m_2}(b_3)_{m_3}}{(c_1)_{m_1}(c_2)_{m_2}(c_3)_{m_3}} \frac{x_1^{m_1}}{m_1!}\frac{x_2^{m_2}}{m_2!}\frac{x_3^{m_3}}{m_3!} \quad (1.1)$$

$$|x_1|+|x_2|+|x_3|<1$$

and

$$F_D^{(3)}(a,b_1,b_2,b_3;d;x_1,x_2,x_3) = \sum_{m_1,m_2,m_3=0}^{\infty} \frac{(a)_{m_1+m_2+m_3}(b_1)_{m_1}(b_2)_{m_2}(b_3)_{m_3}}{(d)_{m_1+m_2+m_3}} \frac{x_1^{m_1}}{m_1!}\frac{x_2^{m_2}}{m_2!}\frac{x_3^{m_3}}{m_3!} \quad (1.2)$$

$$\max\{|x_1|,|x_2|,|x_3|\}<1,$$

where $(a)_n$ is the Pochhammer's symbol defined by

$$(a)_n = \begin{cases} 1 & , \ if \ \ n=0 \\ a(a+1)(a+2)\ldots(a+n-1) & , \ if \ \ n=1,2,3,\ldots\ldots \end{cases} \quad (1.3)$$

The Laplace integral representations ofLauricella functions $F_A^{(3)}$ and $F_D^{(3)}$ are given by Exton (see [3]):

$$F_A^{(3)}(a,b_1,b_2,b_3;c_1,c_2,c_3;x_1,x_2,x_3) = \frac{1}{\Gamma(b_1)\Gamma(b_2)\Gamma(b_3)} \int_0^\infty\int_0^\infty\int_0^\infty e^{-t_1-t_2-t_3}\, t_1^{b_1-1} t_2^{b_2-1} t_3^{b_3-1}$$
$$\times \Psi_2^{(3)}(a;c_1,c_2,c_3;\ x_1t_1,x_2t_2,x_3t_3)\ dt_1 dt_2\ dt_3, \quad (1.4)$$

$$\mathrm{Re}(b_1),\mathrm{Re}(b_2),\mathrm{Re}(b_3)>0$$

and

$$F_D^{(3)}\left(a, b_1, b_2, b_3; d; x_1, x_2, x_3\right)$$

$$= \frac{1}{\Gamma(a)} \int_0^\infty e^{-s} s^{a-1} \Phi_2^{(3)}(b_1, b_2, b_3; d; x_1 s, x_2 s, x_3 s)\, ds, \qquad (1.5)$$

$$\operatorname{Re}(a) > 0,$$

where the functions $\Phi_2^{(3)}$ and $\Psi_2^{(3)}$ are the confluent hypergeometric functions defined by Srivastava[7] .

$$\Phi_2^{(3)}\left(b_1, b_2, b_3; c; x_1, x_2, x_3\right)$$

$$= \sum_{m_1, m_2, m_3 = 0}^{\infty} \frac{(b_1)_{m_1} (b_2)_{m_2} (b_3)_{m_3}}{(c)_{m_1 + m_2 + m_3}} \frac{x_1^{m_1}}{m_1!} \frac{x_2^{m_2}}{m_2!} \frac{x_3^{m_3}}{m_3!} \quad (1.6)$$

and

$$\Psi_2^{(3)}\left(a; b_1, b_2, b_3; x_1, x_2, x_3\right)$$

$$= \sum_{m_1, m_2, m_3 = 0}^{\infty} \frac{(a)_{m_1 + m_2 + m_3}}{(b_1)_{m_1} (b_2)_{m_2} (b_3)_{m_3}} \frac{x_1^{m_1}}{m_1!} \frac{x_2^{m_2}}{m_2!} \frac{x_3^{m_3}}{m_3!}. \qquad (1.7)$$

The Exton’s double hypergeometric functions are defined by [4]

$$X^{A:B;B'}_{C:D;D'}\left[\begin{matrix}(a):(b);(b');\\(c):(d);(d');\end{matrix}\; x, y\right] = \sum_{m,n=0}^{\infty} \frac{((a))_{2m+n}\, ((b))_m\, ((b'))_n}{((c))_{2m+n}\, ((d))_m\, ((d'))_n} \frac{x^m\, y^n}{m!\, n!}, \qquad (1.8)$$

where the symbol $((a))_m$ denotes the product $\prod_{j=1}^{A} (a_j)_m$.

2.MainIntegralFormulas

By employing the generalized Dixon’s theorem [5] and the generalized Kummer’s theorem [6],Atash [1] and Atash and Bellehaj [2] derived the following two extension formulas for Lauricella’s functions of three variables $F_A^{(3)}$ and $F_D^{(3)}$:

$$F_A^{(3)}\left(a, a', b-i, b; d, c, c+i+j; x, y, -y\right)$$

$$= \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \frac{(a)_{m+2n}\, (a')_m\, (b-i)_{2n}\, x^m (-y^2/4)^n}{(d)_m (c)_{2n}\, m! n!}$$

$$\times \frac{2^{2(2n+c-1)+i+j}\, \Gamma(1-b+i-2n)\, \Gamma(c+i+j)\, \Gamma(b-\tfrac{1}{2}|i|-\tfrac{1}{2}i)\Gamma(1-2n-c-\tfrac{1}{2}(i+j+|i+j|)}{\Gamma(b)\Gamma(1-c-2n)\Gamma\left(2c-1+i+j+2n\right)\Gamma(c-b+i+j)}$$

$$\times C_{i,j} \frac{\Gamma(n+c-\tfrac{1}{2}+[\tfrac{i+j+1}{2}])\Gamma(n-b+c+i+[\tfrac{j+1}{2}])}{\Gamma(\tfrac{1}{2})\Gamma(1-b-n+[\tfrac{i}{2}])}$$

$$-\frac{ay}{2} \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \frac{(a+1)_{m+2n}\, (a')_m\, (b-i)_{2n+1}\, x^m (-y^2/4)^n}{(d)_m (c)_{2n+1}\, m! n!}$$

$$\times \frac{2^{2(c+2n)+i+j}\, \Gamma(i-b-2n)\, \Gamma(c+i+j)\, \Gamma(b-\tfrac{1}{2}|i|-\tfrac{1}{2}i)\Gamma(-2n-c-\tfrac{1}{2}(i+j+|i+j|))}{\Gamma(b)\Gamma(-2n-c)\Gamma\left(2c+i+j+2n\right)\Gamma(c-b+i+j)}$$

$$\times D_{i,j} \frac{\Gamma(\tfrac{1}{2}+n+c+[\tfrac{i+j}{2}])\Gamma(1+n-b+c+i+[\tfrac{j}{2}])}{\Gamma(\tfrac{1}{2})\Gamma(-n-b+[\tfrac{i+1}{2}])}, \qquad (2.1)$$

for $i = -3, -2, -1, 0, 1, 2$ and $j = 0, 1, 2, 3$

and

$$F_D^{(3)}\left(a, b-i, b, c; d; x, -x, y\right)$$

$$= \sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\frac{(a)_{2m+n}\ (b-i)_{2m}(c)_n\ x^{2m}\ y^n}{(d)_{2m+n}\ (2m)!\ n!}$$

$$\Bigg\{A_i'\frac{2^{2m}\Gamma(\tfrac{1}{2})\Gamma(1-2m-b+i)\Gamma(1-b)}{\Gamma(1-b+\tfrac{1}{2}(i+|i|))\Gamma(-m+\tfrac{1}{2}i+\tfrac{1}{2}-[\tfrac{1+i}{2}])\Gamma(1-m-b+\tfrac{1}{2}i)}$$

$$+B_i'\frac{2^{2m}\Gamma(\tfrac{1}{2})\Gamma(1-2m-b+i)\Gamma(1-b)}{\Gamma(1-b+\tfrac{1}{2}(i+|i|))\Gamma(-m+\tfrac{1}{2}i-[\tfrac{i}{2}])\Gamma(-m+\tfrac{1}{2}-b+\tfrac{1}{2}i)}\Bigg\}$$

$$+\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\frac{(a)_{2m+n+1}\ (b-i)_{2m+1}(c)_n\ x^{2m+1}\ y^n}{(d)_{2m+n+1}\ (2m+1)!\ n!}$$

$$\times\Bigg\{A_i''\frac{2^{2m+1}\Gamma(\tfrac{1}{2})\Gamma(-2m-b+i)\Gamma(1-b)}{\Gamma(1-b+\tfrac{1}{2}(i+|i|))\Gamma(-m+\tfrac{1}{2}i-[\tfrac{1+i}{2}])\Gamma(-m+\tfrac{1}{2}-b+\tfrac{1}{2}i)}$$

$$+B_i''\frac{2^{2m+1}\Gamma(\tfrac{1}{2})\Gamma(-2m-b+i)\Gamma(1-b)}{\Gamma(1-b+\tfrac{1}{2}(i+|i|))\Gamma(-m-\tfrac{1}{2}+\tfrac{1}{2}i-[\tfrac{i}{2}])\Gamma(-m-b+\tfrac{1}{2}i)}\Bigg\}\quad(2.2)$$

for $i=0,\pm1,\pm2,\pm3,\pm4,\pm5$,

where $[x]$ denotes the greatest integer less than or equal to x and $|x|$ denotes the usual absolute value of x . The coefficient $C_{i,j}$ can be obtained from the table of $A_{i,j}$ given in [5] by replacing a and c by $-2n$ and $1-c-2n$ and the coefficient $D_{i,j}$ can be obtained from the table of $B_{i,j}$ given in [5] by replacing a and c by $-2n-1$ and $-c-2n$ respectively .

The coefficients A'_i and B'_i can be obtained from the tables of A_i and B_i given in [6] by taking $a=-2m$ and the coefficients A''_i and B''_i can be obtained from the same tables of A_i and B_i by taking $a=-2m-1$.

In (1.4) replacing $b_1, b_2, b_3, c_1, c_2, c_3, x_1, x_2$ and x_3 by $a', b-i, b, d, c, c+i+j, x, y$ and $-y$ respectively and using the result (2.1), we get the following general integral involving confluent hypergeometric function of three variables $\Psi_2^{(3)}$:

First Integral

$$\frac{1}{\Gamma(a')\Gamma(b-i)\Gamma(b)}\int_0^{\infty}\int_0^{\infty}\int_0^{\infty}e^{-t_1-t_2-t_3}\ t_1^{a'-1}t_2^{b-i-1}t_3^{b-1}$$

$$\times\Psi_2^{(3)}(a;d,c,c+i+j;xt_1,yt_2,-yt_3)\ dt_1dt_2\,dt_3\,.$$

$$=\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\frac{(a)_{m+2n}\ (a')_m\,(b-i)_{2n}\,x^m(-y^2/4)^n}{(d)_m(c)_{2n}\ m!n!}$$

$$\times\frac{2^{2(2n+c-1)+i+j}\Gamma(1-b+i-2n)\,\Gamma(c+i+j)\,\Gamma(b-\tfrac{1}{2}|i|-\tfrac{1}{2}i)\Gamma(1-2n-c-\tfrac{1}{2}(i+j+|i+j|}{\Gamma(b)\Gamma(1-c-2n)\Gamma(2c-1+i+j+2n)\Gamma(c-b+i+j)}$$

$$\times C_{i,j}\ \frac{\Gamma(n+c-\tfrac{1}{2}+[\tfrac{i+j+1}{2}])\Gamma(n-b+c+i+[\tfrac{j+1}{2}])}{\Gamma(\tfrac{1}{2})\Gamma(1-b-n+[\tfrac{i}{2}])}$$

$$-\frac{ay}{2}\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\frac{(a+1)_{m+2n}\ (a')_m\,(b-i)_{2n+1}x^m(-y^2/4)^n}{(d)_m\,(c)_{2n+1}\ m!n!}$$

$$\times\frac{2^{2(c+2n)+i+j}\,\Gamma(i-b-2n)\,\Gamma(c+i+j)\,\Gamma(b-\tfrac{1}{2}|i|-\tfrac{1}{2}i)\Gamma(-2n-c-\tfrac{1}{2}(i+j+|i+j|)}{\Gamma(b)\Gamma(-2n-c)\Gamma\left(2c+i+j+2n\right)\Gamma(c-b+i+j)}$$

$$\times D_{i,j}\,\frac{\Gamma(\tfrac{1}{2}+n+c+[\tfrac{i+j}{2}])\Gamma(1+n-b+c+i+[\tfrac{j}{2}])}{\Gamma(\tfrac{1}{2})\Gamma(-n-b+[\tfrac{i+1}{2}])}, \qquad (2.3)$$

for $i=-3,-2,-1,0,1,2$ and $j=0,1,2,3$.

Next, in (1.5), replacing b_1, b_2, b_3, x_1, x_2 and x_3 by $b-i, b, c, x, -x$ and y respectively and using the result (2.2), we get the following general integral involving confluent hypergeometric function of three variables $\Phi_2^{(3)}$:

Second Integral

$$\frac{1}{\Gamma(a)}\int_0^\infty e^{-s}s^{a-1}\,\Phi_2^{(3)}(b-i,b,c;\,d;xs,-xs,ys)\,ds$$

$$=\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\frac{(a)_{2m+n}\,(b-i)_{2m}(c)_n\,x^{2m}\,y^n}{(d)_{2m+n}\,(2m)!\,n!}$$

$$\left\{A_i'\frac{2^{2m}\Gamma(\tfrac{1}{2})\Gamma(1-2m-b+i)\Gamma(1-b)}{\Gamma(1-b+\tfrac{1}{2}(i+|i|))\Gamma(-m+\tfrac{1}{2}i+\tfrac{1}{2}-[\tfrac{1+i}{2}])\Gamma(1-m-b+\tfrac{1}{2}i)}\right.$$

$$\left.+B_i'\frac{2^{2m}\Gamma(\tfrac{1}{2})\Gamma(1-2m-b+i)\Gamma(1-b)}{\Gamma(1-b+\tfrac{1}{2}(i+|i|))\Gamma(-m+\tfrac{1}{2}i-[\tfrac{i}{2}])\Gamma(-m+\tfrac{1}{2}-b+\tfrac{1}{2}i)}\right\}$$

$$+\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\frac{(a)_{2m+n+1}\,(b-i)_{2m+1}(c)_n\,x^{2m+1}\,y^n}{(d)_{2m+n+1}\,(2m+1)!\,n!}$$

$$\times\left\{A_i''\frac{2^{2m+1}\Gamma(\tfrac{1}{2})\Gamma(-2m-b+i)\Gamma(1-b)}{\Gamma(1-b+\tfrac{1}{2}(i+|i|))\Gamma(-m+\tfrac{1}{2}i-[\tfrac{1+i}{2}])\Gamma(-m+\tfrac{1}{2}-b+\tfrac{1}{2}i)}\right.$$

$$\left.+B_i''\frac{2^{2m+1}\Gamma(\tfrac{1}{2})\Gamma(-2m-b+i)\Gamma(1-b)}{\Gamma(1-b+\tfrac{1}{2}(i+|i|))\Gamma(-m-\tfrac{1}{2}+\tfrac{1}{2}i-[\tfrac{i}{2}])\Gamma(-m-b+\tfrac{1}{2}i)}\right\}\,(2.4)$$

for $i=0,\pm1,\pm2,\pm3,\pm4,\pm5$.

3. Applications

In this section, we willuse in each case the following results [7]:

$$(a)_n=\frac{\Gamma(a+n)}{\Gamma(a)},\quad a\neq 0,-1,-2,\cdots\ (3.1)$$

$$(a)_{2n}=2^{2n}\left(\tfrac{1}{2}a\right)_n\left(\tfrac{1}{2}a+\tfrac{1}{2}\right)_n \qquad (3.2)$$

$$\frac{\Gamma(a-n)}{\Gamma(a)}=\frac{(-1)^n}{(1-a)_n},\ a\neq 0,\pm1,\pm2,\cdots\ (3.3)$$

$$(2n)!=2^{2n}\left(\tfrac{1}{2}\right)_n n!\ (3.4)$$

$(2n+1)!=2^{2n}\left(\tfrac{3}{2}\right)_n n!$ (3.5)(i) Setting $i=j=0$ in (2.3), we get

$$\frac{1}{\Gamma(a')\Gamma(b)\Gamma(b)}\int_0^\infty\int_0^\infty\int_0^\infty e^{-t_1-t_2-t_3}\, t_1^{a'-1}t_2^{b-1}t_3^{b-1}\,\Psi_2^{(3)}(a;d,c,c;xt_1,yt_2,-yt_3)\ dt_1dt_2\,dt_3$$

$$= X\begin{matrix}1:2;1\\0:3;1\end{matrix}\left[\begin{matrix}a\ :\ \ b,c-b\ \ ;a';\\ -\ :c,\ \tfrac{1}{2}c,\tfrac{1}{2}c+\tfrac{1}{2}\ ;d\,;\end{matrix}\ \frac{y^2}{4},x\right].(3.6)$$

(ii) Setting $i=0,\, j=1$ in (2.3) , we get

$$\frac{1}{\Gamma(a')\Gamma(b)\Gamma(b)}\int_0^\infty\int_0^\infty\int_0^\infty e^{-t_1-t_2-t_3}\, t_1^{a'-1}t_2^{b-1}t_3^{b-1}\,\Psi_2^{(3)}(a;d,c,c+1;xt_1,yt_2,-yt_3)\ dt_1dt_2\,dt_3$$

$$= X\begin{matrix}1:2;1\\0:3;1\end{matrix}\left[\begin{matrix}a:\ \ b,c-b+1\ \ ;a';\\ -:c,\tfrac{1}{2}c+\tfrac{1}{2},\tfrac{1}{2}c+1;d\,;\end{matrix}\ \frac{y^2}{4},x\right]$$

$$+\frac{aby}{c(c+1)}X\begin{matrix}1:2;1\\0:3;1\end{matrix}\left[\begin{matrix}a+1:\ \ b+1,c-b+1\ \ ;a';\\ -\ :c+1,\tfrac{1}{2}c+1,\tfrac{1}{2}c+\tfrac{3}{2};d\,;\end{matrix}\ \frac{y^2}{4},x\right] \qquad .(3.7)$$

(iii) Setting $i=j=1$ in (2.3), we get

$$\frac{1}{\Gamma(a')\Gamma(b-1)\Gamma(b)}\int_0^\infty\int_0^\infty\int_0^\infty e^{-t_1-t_2-t_3}\, t_1^{a'-1}t_2^{b-2}t_3^{b-1}\,\Psi_2^{(3)}(a;d,c,c+2;xt_1,yt_2,-yt_3)\ dt_1dt_2\,dt_3$$

$$= X\begin{matrix}1:2;1\\0:3;1\end{matrix}\left[\begin{matrix}a:\ \ b,c-b+2\ \ ;a';\\ -:c+1,\tfrac{1}{2}c+1,\tfrac{1}{2}c+\tfrac{3}{2};d\,;\end{matrix}\ \frac{y^2}{4},x\right]$$

$$+\frac{a(c-2b+2)y}{c(c+2)}X\begin{matrix}1:2;1\\0:3;1\end{matrix}\left[\begin{matrix}a+1:\ \ b,c-b+2\ \ ;a';\\ -\ :c+1,\tfrac{1}{2}c+\tfrac{3}{2},\tfrac{1}{2}c+2;d\,;\end{matrix}\ \frac{y^2}{4},x\right].\quad (3.8)$$

(iv) Setting $i=0$ in (2.4), we get

$$\frac{1}{\Gamma(a)}\int_0^\infty e^{-s}s^{a-1}\,\Phi_2^{(3)}(b,b,c;d;xs,-xs,ys)\,ds = X\begin{matrix}1:1;1\\1:0;0\end{matrix}\left[\begin{matrix}a:b;c;\\ d:-;-;\end{matrix}\ x^2,y\right]. \qquad (3.9)$$

(v) Setting $i=-1$ in (2.4), we get

$$\frac{1}{\Gamma(a)}\int_0^\infty e^{-s}s^{a-1}\,\Phi_2^{(3)}(b+1,b,c;d;xs,-xs,ys)\ ds$$

$$= X\begin{matrix}1:1;1\\1:0;0\end{matrix}\left[\begin{matrix}a:b+1;c;\\ d:\ -\ ;-;\end{matrix}\ x^2,y\right]+\frac{ax}{d}X\begin{matrix}1:1;1\\1:0;0\end{matrix}\left[\begin{matrix}a+1:b+1;c;\\ d+1:\ -\ ;-;\end{matrix}\ x^2,y\right].(3.10)$$

(vi) Setting $i=1$ in (2.4), we get

$$\frac{1}{\Gamma(a)}\int_0^\infty e^{-s}s^{a-1}\,\Phi_2^{(3)}(b-1,b,c;d;xs,-xs,ys)\ ds$$

$$= X\begin{matrix}1:1;1\\1:0;0\end{matrix}\left[\begin{matrix}a:b;c;\\ d:-;-;\end{matrix}\ x^2,y\right]-\frac{a\,x}{d}X\begin{matrix}1:1;1\\1:0;0\end{matrix}\left[\begin{matrix}a+1:b;c;\\ d+1:-;-;\end{matrix}\ x^2,y\right], (3.11)$$

which for $x=-x$ and $b=b+1$ gives the result (3.10).

The other special cases of (2.3) and (2.4) can be obtained by the similar manner .

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صيغ تكاملية تتضمن الدوال الفوق هندسية ثلاثية المتغيرات $\Phi_2^{(3)}$ و $\Psi_2^{(3)}$

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الملخص

هدف بحثنا هذا هو إثبات صيغ تكاملية عامة متضمنة الدوال الفوق هندسية ثلاثية المتغيرات $\Phi_2^{(3)}$ و $\Psi_2^{(3)}$ وذلك بمساعدة صيغتين لدوال لارسيلا $F_A^{(3)}$ و $F_D^{(3)}$ ذات الثلاثة المتغيرات والمعطاة في [1] و [2] أيضاً تم عرض بعض التطبيقات لنتائج بحثنا الرئيسية.

الكلمات المفتاحية: صيغ تكاملية، الدوال الفوق هندسية، نظرية ديكسون، نظرية كومر.