# Exact solutions for a new models of nonlinear partial differential equations Using $\left(\frac{G'}{G^2}\right)$-Expansion Method

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DOI: https://doi.org/10.47372/uajnas.2019.n1.a16

## Abstract

In this paper, we present a new model of Kadomtsev–Petviashvili (KP) equation, the KadomtsevPetviashvili–equal width (KP-EW) equation and the Yu–Toda–Sassa–Fukuyama (YTSF) equation. We apply the $\left(\frac{G'}{G^2}\right)$-expansion method to solve the new models. Exact travelling wave solutions are obtained and expressed in terms of hyperbolic functions, trigonometric functions, rational functionssolutions of this equations from the method, with the aid of the software Maple.

**Keywords:** Kadomtsev–Petviashvili (KP) equation, modified(KP) equation, Kadomtsev–Petviashvili–equal width (KP-EW) equation, modified (KP-EW) equation, Yu–Toda–Sassa–Fukuyama (YTSF) equation, modified (YTSF) equation, exact solutions,$\left(\frac{G'}{G^2}\right)$-expansion method.

## Introduction

Exact solutions of nonlinear partial differential equations (NLPDEs) play a dynamic role in nonlinear sciences. Numerous techniques have been proposed to investigateexact solutions of such equations (NLPDEs). The detailed study of literature reveals some credible contributions in this area. A variety of many authentic methods have been suggested to get the exact solutions of partial differential equations (PDEs) and have beenexpansion-established such as the$\left(\frac{G'}{G}\right)$ method [9], the tanh-coth method[8,11], the Jacobi elliptic function expansion method [18], Darbouxtransformation [6],the exp-function method [2], the sine-cosine method [4], Bäcklund transformation method [10], the mapping method [12,14] and the multiple soliton solutions[15].

Recently, Li Wen-An, Chen Hao and Zhang Guo-Cai [17] introduced a new approach, namely the $\left(\frac{G'}{G^2}\right)$-expansion method, for a reliable treatment of the nonlinear wave equations. The useful $\left(\frac{G'}{G^2}\right)$-expansion method is then widely used by many authors [3,5,13,17,19].This can be method applied to various nonlinear equations and also gives a few new kinds of solution. Partial differential equations acquired lot of interest and

## Description of the $\left(\frac{G'}{G^2}\right)$-Expansion Method

attracted attention of many studies due to their frequent occurrence in biochemical, mathematics, viscoelasticity, economics and other areas of science.

In this paper, we applied the $\left(\frac{G'}{G^2}\right)$-expansion method to solve the combined Kadomtsev–Petviashvili (KP) equation, combined theKadomtsevPetviashvili-equal width (KP-EW) equation and combined theYu–Toda–Sassa–Fukuyama (YTSF) equation.

Consider the general nonlinear partial differential equations (NLPDEs), say, in two variables,

$P(u, u_x, u_t, u_{xx}, u_{xt}, \dots) = 0,$ (1)

Eq. (1) can be solved by using the following steps:

**Step 1**

Use the wave variable $\xi = k(x - ct)$, where $k$ is the wave number and $c$ is the wave speedto change the PDE (1) in to ODE

$Q(u, u', u'', \dots) = 0.$ (2)

In the above equation, $'$ denotes to the differentiation with respect to $\xi$.

**Step 2**

We suppose that the solution of Eq.(2) has the form

$$u(x,t) = u(\xi) = a_0 + \sum_{i=1}^{n} a_i \left(\frac{G'}{G^2}\right)^i, \quad (3)$$

where the coefficients $a_0$, $a_i$, $k$ and $c$ are constants to be determinedand$\left(\frac{G'}{G^2}\right)$satisfies a nonlinear ordinary differential equation

$$\left(\frac{G'}{G^2}\right)' = \mu + \lambda \left(\frac{G'}{G^2}\right)^2, \quad (4)$$

where$\mu$ and $\lambda$are arbitrary constants, shut that $\mu \neq 1,\ \lambda \neq 0$.

The value of positive integer $n$ is easy to find by balancing the highest order nonlinear terms with the highest order derivative term appearing in Eq. (2).

**Step 3**

Substituting Eq. (3) into Eq.(2) and using Eq. (4), collect the coefficients with the same order of$\left(\frac{G'}{G^2}\right)^i$, $(i = 1, 2, \dots, n)$ and set the coefficients to zero, nonlinear algebraic equations are acquired. Solutions to the resulting algebraic system are derived by using the$\left(\frac{G'}{G^2}\right)$-expansion method with the aid of Maple.

**Step 4**

On the basis of the general solutions to Eq.(4), the ratio $\left(\frac{G'}{G^2}\right)$can be divided into three cases:

**Case 1.** If $\lambda\mu > 0$, then

$$\left(\frac{G'}{G^2}\right) = \sqrt{\frac{\mu}{\lambda}} \left(\frac{k_1 cos(\sqrt{\lambda\mu}\,\xi) + k_2 sin(\sqrt{\lambda\mu}\,\xi)}{k_2 cos(\sqrt{\lambda\mu}\,\xi) - k_1 sin(\sqrt{\lambda\mu}\,\xi)}\right),$$

**Case2.** If $\lambda\mu < 0$, then

$$\left(\frac{G'}{G^2}\right) = \frac{-\sqrt{|\lambda\mu|}}{\lambda} \left(\frac{k_1 cosh\left(2\sqrt{|\lambda\mu|}\,\xi\right) + k_1 sinh\left(2\sqrt{|\lambda\mu|}\,\xi\right) + k_2}{k_1 cosh\left(2\sqrt{|\lambda\mu|}\,\xi\right) + k_1 sinh\left(2\sqrt{|\lambda\mu|}\xi\right) - k_2}\right),$$

**Case3.** If$\mu = 0,\ \lambda \neq 0$, then

$$\left(\frac{G'}{G^2}\right) = -\frac{k_1}{\lambda(k_1\xi + k_2)}.$$

In the above expressions,$k_1$and $k_2$ are nonzero constants. The multiple exact special solutions of nonlinear partial differential equation (1) are obtained by making use of Eq. (3) and the solutions of ODE(4).

## Applications

In this section, we determine the exact traveling wave solutions of the nonlinear (cmKP), (KP-cmEW) and (cmYTSF) equations by using $\left(\frac{G'}{G^2}\right)$-expansion method.

## Exact Solutions for cmKP Equation

We consider a combined(2+1)-dimensional Kadomtsev–Petviashvili (KP) and the modified(2+1)- dimensionalKadomtsev–Petviashvili (mKP) equation as the form

$$(-4v_t + 6p(v)v_x + v_{xxx})_x + 3v_{yy} = 0, \quad v = v(x,y,t), \tag{5}$$
$$P(v) = 2v - v^2 + \partial_x^{-1} v_y,$$

and donated by(cmKP).

Where

$$(-4v_t + 12vv_x + v_{xxx})_x + 3v_{yy} = 0, \tag{6}$$

is the Kadomtsev–Petviashvili (KP) equation [16],

and

$$(-4v_t - 6v^2 v_x + v_{xxx})_x + 3v_{yy} + 6v_x v_y + 6v_{xx}\,\partial_x^{-1} v_y = 0, \tag{7}$$

is the modified Kadomtsev–Petviashvili (mKP) equation [16].

Assuming $u = \partial_x^{-1} v_y$ implies $u_x = v_y$

and using the transformation $v(x,y,t) = v(\xi),\ \xi = k(x + y - ct)$in Eq. (5), we find

$$\left(4cv'(\xi) + 12v(\xi)v'(\xi) - 6\big(v(\xi)\big)^2 v'(\xi) + k^2 v'''(\xi)\right)' + 3v''(\xi) + 6\big(u(\xi)v'(\xi)\big)' = 0,$$
$$u'(\xi) = v'(\xi). \tag{8}$$

Integrating the second equation in the system (8) and neglecting the constants of integration, we find

$$u(\xi) = v(\xi). \tag{9}$$

Substituting Eq. (9) into the first equation of the system (8) and integrating the resulting equation, we find

$$(3 + 4c)v(\xi) + k^2 v''(\xi) + 9\big(v(\xi)\big)^2 - 2\big(v(\xi)\big)^3 = 0. \tag{10}$$

In which $v(\xi), v'(\xi), v''(\xi) \to 0$as $\xi \to \pm\infty$.

Eq. (10) is nonlinear ordinary differential equation.

Balancing the highest order of the nonlinear term $v^3$ with the highest order derivative $v''$ gives $3n = n + 2$ that gives $n = 1$.Now, we apply the $\left(\frac{G'}{G^2}\right)$-expansion method tosolve our equation. Consequently, we get the original solutions for our new equation as the following:

Assume, the solution of Eq. (10) has the form

$$v(x,y,t) = v(\xi) = a_0 + a_1\left(\frac{G'}{G^2}\right), \tag{11}$$

where $a_0$ and $a_1$are constants.

By substituting Eq. (11) in Eq. (10) and using Eq. (4),the left hand side is converted into polynomials in$\left(\frac{G'}{G^2}\right)^i$ , $0 \le i \le 3$.Setting each coefficient of these resulted polynomials to zero, we obtain a set of algebraic equations for $a_0, a_1, c$ and $k$. Solving the system of algebraic equations, with the help of algebraic software Maple, we obtain

Set 1. $a_0 = \frac{3}{2}, a_1 = \frac{3\lambda}{2\sqrt{-\lambda\mu}}, c = -3, k = \pm\frac{3}{2\sqrt{-\lambda\mu}}$, Set 2. $a_0 = \frac{3}{2}, a_1 = \frac{-3\lambda}{2\sqrt{-\lambda\mu}}, c = -3, k = \pm\frac{3}{2\sqrt{-\lambda\mu}}$.

The above set of values yields the following exact solutions cm KP.

From set1: (i) When $\lambda\mu > 0$

$$v_{1,2}(x,y,t) = \frac{3}{2}\left(1 - i\left(\frac{k_1 cosh\left(\frac{\xi}{2}\right) \pm i\, k_2 sinh\left(\frac{\xi}{2}\right)}{k_2 cosh\left(\frac{\xi}{2}\right) \mp i k_1 sinh\left(\frac{\xi}{2}\right)}\right)\right),$$

$$u_{1,2}(x,y,t) = \frac{3}{2}\left(1 - i\left(\frac{k_1 cosh\left(\frac{\xi}{2}\right) \pm i\, k_2 sinh\left(\frac{\xi}{2}\right)}{k_2 cosh\left(\frac{\xi}{2}\right) \mp i k_1 sinh\left(\frac{\xi}{2}\right)}\right)\right),$$

where$\xi = 3(x + y + 3t), k_1, k_2$ , $\mu$ and $\lambda$ are arbitrary constants.

(ii) When $\lambda\mu < 0$

$$v_{3,4}(x,y,t) = \frac{3}{2}\left(1 - \left(\frac{k_1 cosh(\xi) \mp k_1 sinh(\xi) + k_2}{k_1 cosh(\xi) \mp k_1 sinh(\xi) - k_2}\right)\right),$$

$$u_{3,4}(x,y,t) = \frac{3}{2}\left(1 - \left(\frac{k_1 cosh(\xi) \mp k_1 sinh(\xi) + k_2}{k_1 cosh(\xi) \mp k_1 sinh(\xi) - k_2}\right)\right).$$

From set 2: (i) When $\lambda\mu > 0$

$$v_{5,6}(x,y,t) = \frac{3}{2}\left(1 + i\left(\frac{k_1 cosh\left(\frac{\xi}{2}\right) \pm i\, k_2 sinh\left(\frac{\xi}{2}\right)}{k_2 cosh\left(\frac{\xi}{2}\right) \mp i k_1 sinh\left(\frac{\xi}{2}\right)}\right)\right),$$

$$u_{5,6}(x,y,t) = \frac{3}{2}\left(1 + i\left(\frac{k_1 cosh\left(\frac{\xi}{2}\right) \pm i\, k_2 sinh\left(\frac{\xi}{2}\right)}{k_2 cosh\left(\frac{\xi}{2}\right) \mp i k_1 sinh\left(\frac{\xi}{2}\right)}\right)\right).$$

(ii) When $\lambda\mu < 0$

$$v_{7,8}(x,y,t) = \frac{3}{2}\left(1 + \left(\frac{k_1 cosh(\xi) \mp k_1 sinh(\xi) + k_2}{k_1 cosh(\xi) \mp k_1 sinh(\xi) - k_2}\right)\right),$$

$$u_{7,8}(x,y,t) = \frac{3}{2}\left(1 + \left(\frac{k_1 cosh(\xi) \mp k_1 sinh(\xi) + k_2}{k_1 cosh(\xi) \mp k_1 sinh(\xi) - k_2}\right)\right).$$

## Exact Solutions for KP-cmEW Equation

We consider a combined (2+1)-dimensional KadomtsevPetviashvili-equal width (KP-EW) equation and(2+1)-dimensional KadomtsevPetviashvili-modified equalwidth (KP-mEW) equation as the form

$$(u_t + \alpha p(u)u_x + \beta u_{xxt})_x + \gamma u_{yy} = 0, \quad u = u(x,y,t), \tag{12}$$

$p(u) = 2u + 3u^2$, whrer $\alpha, \beta$ and $\gamma$ are real numbers and donated by(KP-cmEW), where

$$(u_t + 2\alpha u u_x + \beta u_{xxt})_x + \gamma u_{yy} = 0, \tag{13}$$

is the KadomtsevPetviashvili-equal width (KP-EW) equation [1] and

$(u_t + 3\alpha u^2 u_x + \beta u_{xxt})_x + \gamma u_{yy} = 0, (14)$

is theKadomtsevPetviashvili-modified equal width (KP-mEW) equation [1].

Using the transformation $u(x,y,t)=u(\xi),\ \xi=k(x+y-ct)$in Eq. (12)and integrating the resulting equation, we find

$$(\gamma-c)u(\xi)-k^2\beta c u''(\xi)+\alpha\big(u(\xi)\big)^2+\alpha\big(u(\xi)\big)^3=0. \qquad (15)$$

In which $u(\xi),u'(\xi),u''(\xi)\to 0$as $\xi\to\pm\infty$.

Eq. (15) is nonlinear ordinary differential equation.

Balancing the highest order of the nonlinear term $u^3$with the highest order derivative $u''$ gives $3n=n+2$ that gives $n=1$.Now, we apply the $\left(\frac{G'}{G^2}\right)$-expansionmethod to solve our equation. Consequently, we get the original solutions for our new equation as the following:

Assume, the solution of Eq. (15) has the form

$$u(x,y,t)=u(\xi)=a_0+a_1\left(\frac{G'}{G^2}\right), \qquad (16)$$

where $a_0$ and $a_1$are constants.

By substituting Eq. (16) in Eq. (15) and using Eq. (4),the left hand side is converted into polynomials in$\left(\frac{G'}{G^2}\right)^i$ , $0\le i\le 3$. Setting each coefficient of these resulted polynomials to zero, we obtain a set of algebraic equations for $a_0,a_1,c$ and $k$. Solving the system of algebraic equations, with the help of algebraic software Maple, we obtain

Set 1: $a_0=\frac{-1}{3},a_1=\frac{1}{3}\sqrt{\frac{-\lambda}{\mu}},c=\frac{-2}{9}\alpha+\gamma,k=\pm\sqrt{\frac{\alpha}{4\alpha\beta\mu\lambda-18\beta\gamma\mu\lambda}}$ ,

Set 2: $a_0=\frac{-1}{3},a_1=\frac{-1}{3}\sqrt{\frac{-\lambda}{\mu}},c=\frac{-2}{9}\alpha+\gamma,k=\pm\sqrt{\frac{\alpha}{4\alpha\beta\mu\lambda-18\beta\gamma\mu\lambda}}$ .

The above set of values yields the following exact solutions KP-cmEW

From set1: $\frac{\alpha}{4\alpha\beta\mu\lambda-18\beta\gamma\mu\lambda}>0$ .

(i) When $\lambda\mu>0$

$$u_{1,2}(x,y,t)=\frac{-1}{3}\left(1-i\left(\frac{k_1\cos\left(\sqrt{\lambda\mu}\,\xi\right)\pm k_2\sin\left(\sqrt{\lambda\mu}\,\xi\right)}{k_2\cos\left(\sqrt{\lambda\mu}\,\xi\right)\mp k_1\sin\left(\sqrt{\lambda\mu}\,\xi\right)}\right)\right),$$

where$\xi=\sqrt{\frac{\alpha}{4\alpha\beta\mu\lambda-18\beta\gamma\mu\lambda}}\left(x+y-\left(\frac{-2}{9}\alpha+\gamma\right)t\right)$, $k_1$, $k_2$ , $\mu$ and $\lambda$ are arbitrary constants.

(ii) When $\lambda\mu<0$

$$u_{3,4}(x,y,t)=\frac{-1}{3}\left(1-\left(\frac{k_1\cosh\left(2\sqrt{|\lambda\mu|}\,\xi\right)\pm k_1\sinh\left(2\sqrt{|\lambda\mu|}\,\xi\right)+k_2}{k_1\cosh\left(2\sqrt{|\lambda\mu|}\,\xi\right)\pm k_1\sinh\left(2\sqrt{|\lambda\mu|}\xi\right)-k_2}\right)\right).$$

From set 2:$\frac{\alpha}{4\alpha\beta\mu\lambda-18\beta\gamma\mu\lambda}>0$ .

(i) When $\lambda\mu>0$

$$u_{5,6}(x,y,t)=\frac{-1}{3}\left(1+i\left(\frac{k_1\cos\left(\sqrt{\lambda\mu}\,\xi\right)\pm k_2\sin\left(\sqrt{\lambda\mu}\,\xi\right)}{k_2\cos\left(\sqrt{\lambda\mu}\,\xi\right)\mp k_1\sin\left(\sqrt{\lambda\mu}\,\xi\right)}\right)\right).$$

(ii) When $\lambda\mu<0$

$$u_{7,8}(x,y,t)=\frac{-1}{3}\left(1+\left(\frac{k_1cosh\left(2\sqrt{|\lambda\mu|}\,\xi\right)\pm k_1sinh\left(2\sqrt{|\lambda\mu|}\,\xi\right)+k_2}{k_1cosh\left(2\sqrt{|\lambda\mu|}\,\xi\right)\pm k_1sinh\left(2\sqrt{|\lambda\mu|}\xi\right)-k_2}\right)\right).$$

From set1: $\frac{\alpha}{4\alpha\beta\mu\lambda-18\beta\gamma\mu\lambda}<0$ .

(i) When $\lambda\mu>0$

$$u_{9,10}(x,y,t)=\frac{-1}{3}\left(1-i\left(\frac{k_1cosh\left(\sqrt{\lambda\mu}\,\xi\right)\pm i\,k_2sinh\left(\sqrt{\lambda\mu}\,\xi\right)}{k_2cosh\left(\sqrt{\lambda\mu}\,\xi\right)\mp i\,k_1sinh\left(\sqrt{\lambda\mu}\,\xi\right)}\right)\right),$$

where$\xi=\sqrt{\frac{-\alpha}{4\alpha\beta\mu\lambda-18\beta\gamma\mu\lambda}}\left(x+y-\left(\frac{-2}{9}\alpha+\gamma\right)t\right)$, $k_1,\ k_2$ , $\mu$ and $\lambda$ are arbitrary constants.

(ii) When $\lambda\mu<0$

$$u_{11,12}(x,y,t)=\frac{-1}{3}\left(1-\left(\frac{k_1cos\left(2\sqrt{|\lambda\mu|}\,\xi\right)\pm i\,k_1sin\left(2\sqrt{|\lambda\mu|}\,\xi\right)+k_2}{k_1cos\left(2\sqrt{|\lambda\mu|}\,\xi\right)\pm i\,k_1sin\left(2\sqrt{|\lambda\mu|}\xi\right)-k_2}\right)\right).$$

From set 2: $\frac{\alpha}{4\alpha\beta\mu\lambda-18\beta\gamma\mu\lambda}<0.$

(i) When $\lambda\mu>0$

$$u_{13,14}(x,y,t)=\frac{-1}{3}\left(1+i\left(\frac{k_1cosh\left(\sqrt{\lambda\mu}\,\xi\right)\pm i\,k_2sinh\left(\sqrt{\lambda\mu}\,\xi\right)}{k_2cosh\left(\sqrt{\lambda\mu}\,\xi\right)\mp i\,k_1sinh\left(\sqrt{\lambda\mu}\,\xi\right)}\right)\right).$$

(ii) When $\lambda\mu<0$

$$u_{15,16}(x,y,t)=\frac{-1}{3}\left(1+\left(\frac{k_1cos\left(2\sqrt{|\lambda\mu|}\,\xi\right)\pm i\,k_1sin\left(2\sqrt{|\lambda\mu|}\,\xi\right)+k_2}{k_1cos\left(2\sqrt{|\lambda\mu|}\,\xi\right)\pm i\,k_1sin\left(2\sqrt{|\lambda\mu|}\xi\right)-k_2}\right)\right).$$

**Exact Solutions for cmYTSF Equation**

We consider a combined (3+1)- dimensional Yu–Toda–Sassa–Fukuyama (YTSF) equation and the modified (3+1)- dimensional Yu–Toda–Sassa–Fukuyama (mYTSF) equation as the form

$$-4u_t+u_{xxz}+4p(u)u_z+2u_x\,\partial_x^{-1}p'(u)u_z+3\,\partial_x^{-1}u_{yy}=0, u=u(x,y,z,t) \qquad (17)$$

where$p(u)=u+u^2$and donated by (cmYTSF ),

where

$$-4u_t+u_{xxz}+4uu_z+2u_x\,\partial_x^{-1}u_z+3\,\partial_x^{-1}u_{yy}=0\ , \qquad (18)$$

is the Yu–Toda–Sassa–Fukuyama (YTSF) equation [7]

and

$$-4u_t+u_{xxz}+4u^2u_z+4u_x\,\partial_x^{-1}uu_z+3\,\partial_x^{-1}u_{yy}=0, \qquad (19)$$

isthe modified Yu–Toda–Sassa–Fukuyama (YTSF) equation.

Assuming $v=\partial_x^{-1}(1+2u)u_z$ and $w=\partial_x^{-1}u_{yy}$ implies $v_x=(1+2u)u_z$ and $w_x=u_{yy}$ , then Eq. (17) reduce to the system

$$-4u_t+u_{xxz}+4(u+u^2)u_z+2u_xv+3w=0,$$
$$v_x=(1+2u)u_z\,,\ w_x=u_{yy}\,. \qquad (20)$$

Using the transformation $u(x,y,z,t)=u(\xi),\ \xi=k(x+y+z-ct)$ in Eq. (20),

$$4cu'(\xi)+k^2u'''(\xi)+4\left(u(\xi)+\left(u(\xi)\right)^2\right)u'(\xi)+2u'(\xi)v(\xi)+3w(\xi)=0,$$

$v'(\xi) = (1+2u)u'(\xi),\ w'(\xi) = u''(\xi).$ (21)

Integrating the second equation in the system (21) and neglecting the constants of integration, we find

$v(\xi) = u(\xi) + \left(u(\xi)\right)^2, \qquad w(\xi) = u'(\xi)$ (22)

Substituting Eq. (22) into the first equation of the system (21) and integrating the resulting equation, we find

$(3+4c)u(\xi) + k^2 u''(\xi) + 3\left(u(\xi)\right)^2 + 2\left(u(\xi)\right)^3 = 0.$ (23)

In which $u(\xi), u'(\xi), u''(\xi) \to 0$as $\xi \to \pm\infty$.

Eq. (23) is nonlinear ordinary differential equation.

Balancing thehighest order of the nonlinear term $u^3$ with the highestorderderivative $u''$gives$3n = n+2$ that gives $n = 1$.Now, we apply the $\left(\frac{G'}{G^2}\right)$-expansionmethod to solve our equation. Consequently, we get the original solutions for our new equation as the following:

Assume, the solution of Eq. (23) has the form

$$u(x,y,z,t) = u(\xi) = a_0 + a_1\left(\frac{G'}{G^2}\right), \tag{24}$$

where $a_0$ and $a_1$are constants.

By substituting Eq. (24) in Eq. (23) and using Eq. (4),the left hand side is converted into polynomials in$\left(\frac{G'}{G^2}\right)^i$ , $0 \le i \le 3$. Setting each coefficient of these resulted polynomials to zero, we obtain a set of algebraic equations for $a_0, a_1, c$ and $k$. Solving the system of algebraic equations, with the help of algebraic software Maple, we obtain

Set 1: $a_0 = \frac{-1}{2}, a_1 = \frac{1}{2}\sqrt{\frac{-\lambda}{\mu}}, c = \frac{-1}{2}, k = \pm\frac{1}{2\sqrt{\lambda\mu}}$ ,

Set 2: $a_0 = \frac{-1}{2}, a_1 = \frac{-1}{2}\sqrt{\frac{-\lambda}{\mu}}, c = \frac{-1}{2}, k = \pm\frac{1}{2\sqrt{\lambda\mu}}$ .

The above set of values yields the following exact solutions cmYTSF.

From set1:(i) When $\lambda\mu > 0$

$$u_{1,2}(x,y,z,t) = \frac{-1}{2}\left(1 - i\left(\frac{k_1 cos\left(\frac{\xi}{2}\right) \pm k_2 sin\left(\frac{\xi}{2}\right)}{k_2 cos\left(\frac{\xi}{2}\right) \mp k_1 sin\left(\frac{\xi}{2}\right)}\right)\right),$$

$$v_{1,2}(x,y,z,t) = \frac{-1}{4}\left(1 + \left(\frac{k_1 cos\left(\frac{\xi}{2}\right) \pm k_2 sin\left(\frac{\xi}{2}\right)}{k_2 cos\left(\frac{\xi}{2}\right) \mp k_1 sin\left(\frac{\xi}{2}\right)}\right)^2\right),$$

$$w_{1,2}(x,y,z,t) = \frac{i}{4}\left(1 + \left(\frac{k_1 cos\left(\frac{\xi}{2}\right) \pm k_2 sin\left(\frac{\xi}{2}\right)}{k_2 cos\left(\frac{\xi}{2}\right) \mp k_1 sin\left(\frac{\xi}{2}\right)}\right)^2\right),$$

where$\xi = \left(x+y+z+\frac{1}{2}t\right), k_1,\ k_2\ ,\ \mu$ and $\lambda$are arbitrary constants.

(ii) When $\lambda\mu < 0$.

$$u_{3,4}(x,y,z,t)=\frac{-1}{2}\left(1-\left(\frac{k_1cos(\xi)\pm\ i\,k_1sin(\xi)+k_2}{k_1cos(\xi)\pm ik_1sin(\xi)-k_2}\right)\right),$$

$$v_{3,4}(x,y,z,t)=\frac{-1}{4}\left(1-\left(\frac{k_1cos(\xi)\pm i\,k_1sin(\xi)+k_2}{k_1cos(\xi)\pm i\,k_1sin(\xi)-k_2}\right)^2\right),$$

$$w_{3,4}(x,y,z,t)=k_1k_2\left(\frac{-i\,cos(\xi)\pm\ sin(\xi)}{(k_1cos(\xi)\pm i\,k_1sin(\xi)-k_2)^2}\right).$$

From set 2: (i) When $\lambda\mu>0$.

$$u_{5,6}(x,y,z,t)=\frac{-1}{2}\left(1+i\left(\frac{k_1cos\left(\frac{\xi}{2}\right)\pm k_2sin\left(\frac{\xi}{2}\right)}{k_2cos\left(\frac{\xi}{2}\right)\mp k_1sin\left(\frac{\xi}{2}\right)}\right)\right),$$

$$v_{5,6}(x,y,z,t)=\frac{-1}{4}\left(1+\left(\frac{k_1cos\left(\frac{\xi}{2}\right)\pm k_2sin\left(\frac{\xi}{2}\right)}{k_2cos\left(\frac{\xi}{2}\right)\mp k_1sin\left(\frac{\xi}{2}\right)}\right)^2\right),$$

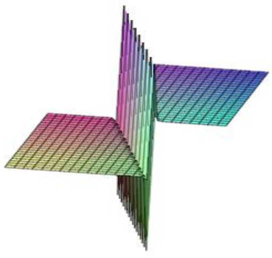
$$w_{5,6}(x,y,z,t)=-\frac{i}{4}\left(1+\left(\frac{k_1cos\left(\frac{\xi}{2}\right)\pm k_2sin\left(\frac{\xi}{2}\right)}{k_2cos\left(\frac{\xi}{2}\right)\mp k_1sin\left(\frac{\xi}{2}\right)}\right)^2\right).$$

(ii) When $\lambda\mu<0$.

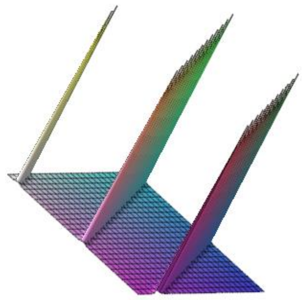
$$u_{7,8}(x,y,z,t)=\frac{-1}{2}\left(1+\left(\frac{k_1cos(\xi)\pm\ i\,k_1sin(\xi)+k_2}{k_1cos(\xi)\pm ik_1sin(\xi)-k_2}\right)\right),$$

$$v_{7,8}(x,y,z,t)=\frac{-1}{4}\left(1-\left(\frac{k_1cos(\xi)\pm i\,k_1sin(\xi)+k_2}{k_1cos(\xi)\pm i\,k_1sin(\xi)-k_2}\right)^2\right),$$

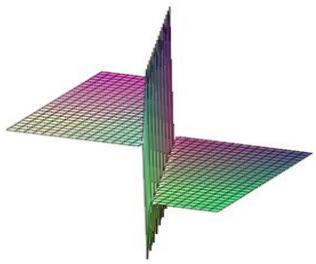
$$w_{7,8}(x,y,z,t)=-k_1k_2\left(\frac{-i\,cos(\xi)\pm\ sin(\xi)}{(k_1cos(\xi)\pm i\,k_1sin(\xi)-k_2)^2}\right).$$



$v_{3,4}(x,y,t)$ when $k_1=1,\ k_2=2$



$v_{1,2}(x,y,z,t)$ when $k_1=1,\ k_2=2$



$v_{7,8}(x,y,t)$ when $k_1=1,\ k_2=2$

## Conclusions

In this paper, the $\left(\frac{G'}{G^2}\right)$-expansion method has been successfully implemented to find new traveling waves solutions for our new proposed equations, namely a combined Kadomtsev–Petviashvili (KP) equation, modified Kadomtsev–Petviashvili (mKP) equation, a combined KadomtsevPetviashvili-equal width (KP-EW) equation, modified KadomtsevPetviashvili-equal width (KP-mEW) equation, a combined Yu–Toda–Sassa–

Fukuyama (YTSF) equation and modified Yu–Toda–Sassa–Fukuyama (mYTSF) equation. The results show that this method is a powerful mathematical tool for obtaining exact solutions for our equations. It is also a promising method to solve other nonlinear partial differential equations.

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# الحلول الدقيقة لنماذج جديدة من المعادلات التفاضلية الجزئية غير الخطية باستخدام طريقة $\left(\frac{G'}{G^2}\right)$ الموسعة

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DOI: https://doi.org/10.47372/uajnas.2019.n1.a16

## الملخص

في هذا البحث قدمنا نموذجاً جديداً لمعادلة كدمسيف بيفي اشيفلي(KP)، معادلة كدمسيف بيفي اشيفلي– اكول ودث (KP-EW) و معادلة يو- تودا- ساسا- فوكوياما (YTSF). ثم قمنا بتطبيق طريقة$\left(\frac{G'}{G^2}\right)$الموسعة لحل النماذج الجديدة للمعادلات المقدمة. إذ تم الحصول على العديد من الحلول الدقيقة للموجات المتنقلة التي تم التعبير عنها بواسطة الدوال الزائدية، الدوال المثلثية والدوال الكسرية لهذه المعادلات وذلك بمساعدة برنامج Maple.

أظهرت النتائج أنً هذه الطريقة هي أداة رياضية قوية للحصول على حلول دقيقة لمعادلاتنا. وهي أيضاً طريقة واعدة لحل المعادلات الجزئية غير الخطية الأخرى.

**الكلمات المفتاحية:** معادلة كدمسيف بيفي اشيفلي(KP) ، معادلة كدمسيف بيفي اشيفلي المعدلة (mKP)، معادلة كدمسيف بيفي اشيفلي– اكول ودث (KP-EW)، معادلة كدمسيف بيفي اشيفلي– اكول ودث المعدلة (KP-mEW)، معادلة يو- تودا- ساسا- فوكوياما (YTSF)، معادلة يو- تودا- ساسا- فوكوياما المعدلة (mYTSF)، الحلول الدقيقة و طريقة $\left(\frac{G'}{G^2}\right)$ الموسعة.