The Generalized Riccati Equation Mapping for Solving (cmZKB) and (pZK) Equations

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Abstract

The generalized Riccati equation mapping is extended which is powerful and straight for Word mathematical tool for solving nonlinear partial differential equations.

In this paper, we construct twenty-seven traveling wave solutions for Combined (1+3) Zakharov-Kuznetsov-burgers equation (cmZKB) and potential (1+3) Zakharov-Kuznetsov Equation (Pzk)by applying this method. In this method $Q' = l + nQ + mQ^2$, is used, as the auxiliary equation, called the generalized Riccati equation, where *l*, *m* and *n* are arbitrary constants. Further, the solutions are expressed in terms of the hyperbolic function, the trigonometric function and elliptic function.

Keywords: The generalized Riccati equation,Combined the (1+3) Zakharov-Kuznetsov-burgers equation, Nonlinear partial differential equations.

Introduction

The study of exact traveling wave solutions for the nonlinear partial differential equations (NPDEs) is one of the attractive and remarkable research fields in all areas of science and engineering, such as plasma physics, chemical physics, optical fibres, chemistry and many others. In the recent years, many researchers implemented various methods to study different nonlinear differential equations for searching traveling wave solutions, for example, the tanh-coth method [9], the Exp-function method[5],the Inverse scattering method[2], the Inverse scattering transform method [1], the Hirota's bilinear methed [6], the painleve expansion method[12] the G'/G – expansion method [11], the generalized Riccati equation mapping method [16] and others. In the present paper, we shall use the improved Riccati equation mapping method to find the exact solutions of (cmZKB) and (Pzk) equations.

The Extended Generalized Riccati Equation Mapping Method

Suppose the general nonlinear partial differential equation:

$$H\left(v, v_t, v_x, v_y, v_{xt}, v_{yt}, v_{xy}, v_{tt}, v_{xx}, v_{yy}, \ldots\right) = 0, \qquad (1)$$

where $v = v(x, y, t)$is an unknown function, H is a polynomial in$v(x, y, t)$and the subscripts indicate the partial derivatives.

The most important steps of the generalized Riccati equation mapping method are as follows:

Step 1:

Consider the traveling wave variable:

$$v(\mathrm{x},\mathrm{y},\mathrm{t}) = v(\beta), \quad \beta = \lambda(\mathrm{x}+\mathrm{y}-\mathrm{ct}), \qquad (2)$$

whereλ and $c$are constant, then Eq. (1) reduces to a nonlinear ordinary differential equation (NODE).

$$F\,(v, v', v'', \ldots) = 0, \qquad (3)$$

where the superscripts stand for the ordinary derivatives with respect toβ.

Step 2:

We suppose that the solution of the ODE (3) can be expressed as follows:

$$V(\beta) = \sum_{i=0}^{r} a_i Q^i(\beta), \qquad (4)$$

where a_i is constant to be determined later such as $a_r \neq 0$ or $a_{-r} \neq 0$ and $Q = Q(\beta)$ is the solution of generalized Riccati equation

$Q' = n + lQ + mQ^2$ (5)

where n, l and m are constants, such that $r \neq 0$.

Step 3:

We determine the positive integer r in Eq. (4) by highest order with the highest order derivative term of $v(\beta)$ in Eq.(3).

Step 4:

Substituting Eq. (4) and along with Eq.(5) into Eq.(3) and setting all the coefficients of Q^i to zero, yield a system of algebraic equations which can be solved by using the Maple to find the values of the constants a_i, c and λ.

Step 5:

We have the following twenty-seven solutions, including four different types solution of Eq.(5).

Family 2.1:

When $\Delta = l^2 - 4mn > 0$ *and* $lm \neq 0$ or $mn \neq 0$, the soluations of Eq. (5) are:

$$Q_1 = \frac{-1}{2m}\left(l + \sqrt{\Delta}\tanh\left(\frac{\sqrt{\Delta}}{2}\beta\right)\right),$$

$$Q_2 = \frac{-1}{2m}\left(l + \sqrt{\Delta}\coth\left(\frac{\sqrt{\Delta}}{2}\beta\right)\right),$$

$$Q_3 = \frac{-1}{2m}\left(l + \sqrt{\Delta}\left(\tanh(\sqrt{\Delta}\beta) \pm i\,\mathrm{sech}(\sqrt{\Delta}\beta)\right)\right), \qquad i = \sqrt{-1}$$

$$Q_4 = \frac{-1}{2m}\left(l + \sqrt{\Delta}\left(\coth(\sqrt{\Delta}\beta) \pm \mathrm{csch}(\sqrt{\Delta}\beta)\right)\right),$$

$$Q_5 = \frac{-1}{4m}\left(2l + \sqrt{\Delta}\left(\tanh\left(\frac{\sqrt{\Delta}}{4}\beta\right) + \coth\left(\frac{\sqrt{\Delta}}{4}\beta\right)\right)\right),$$

$$Q_6 = \frac{1}{2m}\left(-l + \frac{\sqrt{(M^2+N^2)\Delta} - M\sqrt{\Delta}\cosh(\sqrt{\Delta}\beta)}{M\cosh(\sqrt{\Delta}\beta) + N}\right),$$

$$Q_7 = \frac{1}{2m}\left(-l - \frac{\sqrt{(N^2-M^2)\Delta} + M\sqrt{\Delta}\sinh(\sqrt{\Delta}\beta)}{M\cosh(\sqrt{\Delta}\beta) + N}\right),$$

where M and N are two nonzero real constants and satisfy $N^2 - M^2 > 0$.

$$Q_8 = \frac{2n\cosh\left(\frac{\sqrt{\Delta}}{2}\beta\right)}{\sqrt{\Delta}\sinh\left(\frac{\sqrt{\Delta}}{2}\beta\right) - l\cosh\left(\frac{\sqrt{\Delta}}{2}\beta\right)},$$

$$Q_9 = \frac{-2n\sinh\left(\frac{\sqrt{\Delta}}{2}\beta\right)}{l\sinh\left(\frac{\sqrt{\Delta}}{2}\beta\right) - \sqrt{\Delta}\cosh\left(\frac{\sqrt{\Delta}}{2}\beta\right)},$$

$$Q_{10} = \frac{2n\cosh(\sqrt{\Delta}\beta)}{\sqrt{\Delta}\sinh(\sqrt{\Delta}\beta) - l\cosh(\sqrt{\Delta}\beta) \pm i\sqrt{\Delta}},$$

$$Q_{11} = \frac{2n\sinh\left(\frac{\sqrt{\Delta}}{2}\beta\right)}{-l\sinh\left(\frac{\sqrt{\Delta}}{2}\beta\right) + \sqrt{\Delta}\cosh\left(\frac{\sqrt{\Delta}}{2}\beta\right) \pm \sqrt{\Delta}},$$

$$Q_{12} = \frac{4n\sinh\left(\frac{\sqrt{\Delta}}{4}\beta\right)\cosh\left(\frac{\sqrt{\Delta}}{4}\beta\right)}{-2l\sinh\left(\frac{\sqrt{\Delta}}{4}\beta\right)\cosh\left(\frac{\sqrt{\Delta}}{4}\beta\right) + 2\sqrt{\Delta}\cosh^2\left(\frac{\sqrt{\Delta}}{4}\beta\right) - \sqrt{\Delta}},$$

Family 2.2:

When$\Delta= l^2 - 4mn < 0$ and $lm \neq 0$ or $mn \neq 0$, the soluations of Eq. (5) are:

$$Q_{13} = \frac{1}{2m}\left(-l + \sqrt{-\Delta}\tan\left(\frac{\sqrt{-\Delta}}{2}\beta\right)\right),$$

$$Q_{14} = \frac{-1}{2m}\left(l + \sqrt{-\Delta}\cot\left(\frac{\sqrt{-\Delta}}{2}\beta\right)\right),$$

$$Q_{15} = \frac{1}{2m}\left(-l + \sqrt{-\Delta}\left(\tan\left(\sqrt{-\Delta}\beta\right) \pm \sec\left(\sqrt{-\Delta}\beta\right)\right)\right),$$

$$Q_{16} = \frac{-1}{2m}\left(l + \sqrt{-\Delta}\left(\cot\left(\sqrt{-\Delta}\beta\right) \pm \csc\left(\sqrt{-\Delta}\beta\right)\right),$$

$$Q_{17} = \frac{1}{4m}\left(-2l + \sqrt{-\Delta}\left(\tan\left(\frac{\sqrt{-\Delta}}{4}\beta\right) - \cot\left(\frac{\sqrt{-\Delta}}{4}\beta\right)\right)\right),$$

$$Q_{18} = \frac{1}{2m}\left(-l + \frac{\sqrt{-\Delta(M^2 - N^2)} - M\sqrt{-\Delta}\cos\left(\sqrt{-\Delta}\beta\right)}{M\sin\left(\sqrt{-\Delta}\beta\right) + N}\right),$$

$$Q_{19} = \frac{1}{2m}\left(-l - \frac{\sqrt{-\Delta(M^2 - N^2)} + M\sqrt{-\Delta}\cos\left(\sqrt{-\Delta}\beta\right)}{M\sin\left(\sqrt{-\Delta}\beta\right) + N}\right),$$

where M and N are two nonzero real constants and satisfy $N^2 - M^2 > 0$.

$$Q_{20} = \frac{-2n\cos\left(\frac{\sqrt{-\Delta}}{2}\beta\right)}{\sqrt{-\Delta}\sin\left(\frac{\sqrt{-\Delta}}{2}\beta\right) - l\cos\left(\frac{\sqrt{-\Delta}}{2}\beta\right)},$$

$$Q_{21} = \frac{2n\sin\left(\frac{\sqrt{-\Delta}}{2}\beta\right)}{-l\sin\left(\frac{\sqrt{-\Delta}}{2}\beta\right) + \sqrt{-\Delta}\cos\left(\frac{\sqrt{-\Delta}}{2}\beta\right)},$$

$$Q_{22} = \frac{-2n\cos\left(\sqrt{-\Delta}\beta\right)}{\sqrt{-\Delta}\sin\left(\sqrt{-\Delta}\beta\right) + l\cos\left(\sqrt{-\Delta}\beta\right) \pm i\sqrt{-\Delta}},$$

$$Q_{23} = \frac{2n\sin\left(\sqrt{-\Delta}\beta\right)}{-l\sin\left(\sqrt{-\Delta}\beta\right) + \sqrt{-\Delta}\cos\left(\sqrt{-\Delta}\beta\right) \pm \sqrt{-\Delta}},$$

$$Q_{24} = \frac{4n\sin\left(\frac{\sqrt{-\Delta}}{4}\beta\right)\cos\left(\frac{\sqrt{-\Delta}}{4}\beta\right)}{-2l\sin\left(\frac{\sqrt{-\Delta}}{4}\beta\right)\cos\left(\frac{\sqrt{-\Delta}}{4}\beta\right) + 2\sqrt{-\Delta}\cos^2\left(\frac{\sqrt{-\Delta}}{4}\beta\right) - \sqrt{-\Delta}},$$

Family 2.3:

When $n = 0$, and $lm \neq 0$, the solutions of eq.(5) become:

$$Q_{25} = \frac{-ld}{m(d + \cosh(l\beta) - \sinh(l\beta))},$$

$$Q_{26} = \frac{-l(\cosh(l\beta) + \sinh(l\beta))}{m(d + \cosh(l\beta) + \sinh(l\beta))},$$

where$d$ is an arbitrary constant.

Family 2.4:

When $m \neq 0$and $n = l = 0$, solution of Eq.(5) become:

$$Q_{27} = \frac{-1}{m\beta + p_1},$$

wherep_1 is an arbitrary constant.

The Combined (1+3) Zakharov- Kuznetsov-Burgers Equation Application.

In this section, we present our proposed equation, namely combinedthe(1+3)-Zakharov-Kuznetsov-Burgers (ZKB) and modified the (1+3)-Zakharov-Kuznetsov -Burgers (mZKB)equations as the form:

$$v_t + v_x - v_{xx} + \big(p(v)\big)v_x + v_{xxx} + v_{xyy} + v_{xzz} = 0, \qquad (6)$$

$$P\,(v) = v + v^2, \qquad \text{where } v = v(x, y, z, t),$$

and donated by (cZKB) ,

where

$$v_t + v_x - v_{xx} + v v_x + v_{xxx} + v_{xyy} + v_{xzz} = 0, \qquad (7)$$

is the (1+3)-Zakharov-Kuznetsov-Burgers (ZKB) equation,

and

$$v_t + v_x - v_{xx} + v^2 v_x + v_{xxx} + v_{xyy} + v_{xzz} = 0, \qquad (8)$$

isthe modified (1+3)-Zakharov-Kuznetsov-Burgers (mZKB) equation.

Now, we apply the improved generalized Riccatiequation mapping method to find many families of exact traveling wave solution of Eq. (6).

To the end, we use the wave transformationof Eq. (2), in Eq. (5) and integrating once yields

$$(1-c)v - \lambda v' + \frac{v^2}{2} + \frac{v^3}{3} + 3\lambda^2 v'' = 0, \qquad (9)$$

balancing the highest order of the nonlinear term $v^3$with the highest order derivative v'' $3r = r + 2$,that gives $r = 1$.

Hence, the formal solution of Eq. (9) takes the form:

$$v\,(\beta) = a_0 + a_1 Q, \qquad (10)$$

wherea_0 and a_1 are constant to be determined, inserting Eq. (10) with the aid of Eq. (5) into Eq. (9)and solving the resulting system, using maple program weobtain the following solution.

$$a_0 = \pm\frac{i\big(18l\lambda \pm 3i\sqrt{2} - 2\big)\sqrt{2}}{12}, \quad a_1 = \pm 3i\sqrt{2}m\lambda, \quad c = \frac{\pm 135 i\sqrt{2} + 146}{\pm 54\big(3i\sqrt{2} + 4\big)}, \quad l = l\,,$$

$$\lambda = \lambda, m = m, \quad n = \frac{\pm 486 i\sqrt{2}\lambda^2 l^2 + 648\lambda^2 l^2 \pm 45 i\sqrt{2} - 8}{648(m\lambda^2(\pm 3i\sqrt{2} + 4)}.$$

Using Eq. (10), the solutions of Eq. (9).

Family 3.1:

When$\Delta = l^2 - 4mn > 0$ and $lm \neq 0$ or $mn \neq 0$. In Eq. (10), we compensate for the values of a_0, a_1 and Q_1 in family 2.1, the solutions of Eq. (6)are given by:

$$v_{1,2} = a_0 \pm 3i\sqrt{2}m\lambda\left(\frac{-1}{2m}\left(l + \sqrt{\Delta}\left(\tanh\left(\frac{\sqrt{\Delta}}{2}\lambda\sigma\right)\right)\right)\right),$$

Simplifying, we get

$$v_{1,2} = a_0 \mp \frac{3i\sqrt{2}\lambda}{2}\left(l + \sqrt{\Delta}\left(\tanh\left(\frac{\sqrt{\Delta}}{2}\lambda\sigma\right)\right)\right),$$

similarly, we find other solutions,

$$v_{3,4} = a_0 \mp \frac{3i\sqrt{2}\lambda}{2}\left(l + \sqrt{\Delta}\left(\coth\left(\frac{\sqrt{\Delta}}{2}\lambda\sigma\right)\right)\right),$$

$$v_{5,6} = a_0 \mp \frac{3i\sqrt{2}\lambda}{2}\left(l + \sqrt{\Delta}\left(\tanh\big(\sqrt{\Delta}\lambda\sigma\big) + i sech\big(\sqrt{\Delta}\lambda\sigma\big)\right)\right),$$

$$v_{7,8} = a_0 \mp \frac{3i\sqrt{2}\lambda}{2}\left(l + \sqrt{\Delta}\left(\coth\big(\sqrt{\Delta}\lambda\sigma\big) + csch\big(\sqrt{\Delta}\lambda\sigma\big)\right)\right),$$

$$v_{9,10} = a_0 \mp \frac{3i\sqrt{2}\lambda}{4}\left(2l + \sqrt{\Delta}\left(\tanh\left(\frac{\sqrt{\Delta}}{4}\lambda\sigma\right) + \coth\left(\frac{\sqrt{\Delta}}{4}\lambda\sigma\right)\right)\right),$$

$$v_{11,12} = a_0 \pm \frac{3i\sqrt{2}\lambda}{2}\left(-l + \frac{\sqrt{\Delta(\mathrm{M}^2+\mathrm{N}^2)} - \mathrm{M}\sqrt{\Delta}\cosh(\sqrt{\Delta}\lambda\sigma)}{\mathrm{M}\sinh(\sqrt{\Delta}\lambda\sigma) + N}\right),$$

$$v_{13,14} = a_0 \pm \frac{3i\sqrt{2}\lambda}{2}\left(-l - \frac{\sqrt{\Delta(\mathrm{N}^2-\mathrm{M}^2)} - \mathrm{M}\sqrt{\Delta}\sinh(\sqrt{\Delta}\lambda\sigma)}{\mathrm{M}\cosh(\sqrt{\Delta}\lambda\sigma) + N}\right),$$

$$v_{15,16} = a_0 \pm 3i\sqrt{2}\left(\frac{(\pm 486i\sqrt{2}\lambda^2 l^2 + 648\lambda^2 l^2 \pm 45i\sqrt{2} - 8)\cosh\left(\frac{\sqrt{\Delta}}{2}\lambda\sigma\right)}{324\lambda(\pm 3i\sqrt{2}+4)\left(\sqrt{\Delta}\sinh\left(\frac{\sqrt{\Delta}}{2}\lambda\sigma\right) - l\cosh\left(\frac{\sqrt{\Delta}}{2}\lambda\sigma\right)\right)}\right),$$

$$v_{17,18} = a_0 \mp 3i\sqrt{2}\left(\frac{(\pm 486i\sqrt{2}\lambda^2 l^2 + 648\lambda^2 l^2 \pm 45i\sqrt{2} - 8)\sinh\left(\frac{\sqrt{\Delta}}{2}\lambda\sigma\right)}{324\lambda(\pm 3i\sqrt{2}+4)\left(l\sinh\left(\frac{\sqrt{\Delta}}{2}\lambda\sigma\right) - \sqrt{\Delta}\cosh\left(\frac{\sqrt{\Delta}}{2}\lambda\sigma\right)\right)}\right),$$

$$v_{19,20} = a_0 \pm 3i\sqrt{2}\left(\frac{(\pm 486i\sqrt{2}\lambda^2 l^2 + 648\lambda^2 l^2 \pm 45i\sqrt{2} - 8)\cosh(\sqrt{\Delta}\lambda\sigma)}{324\lambda(\pm 3i\sqrt{2}+4)(\sqrt{\Delta}\sinh(\sqrt{\Delta}\lambda\sigma) - l\cosh(\sqrt{\Delta}\lambda\sigma)) + i\sqrt{\Delta}}\right),$$

$$v_{21,22} = a_0 \mp 3i\sqrt{2}\left(\frac{(\pm 486i\sqrt{2}\lambda^2 l^2 + 648\lambda^2 l^2 \pm 45i\sqrt{2} - 8)\sinh(\sqrt{\Delta}\lambda\sigma)}{324\lambda(\pm 3i\sqrt{2}+4)(l\sinh(\sqrt{\Delta}\lambda\sigma) - \sqrt{\Delta}\cosh(\sqrt{\Delta}\lambda\sigma)) + \sqrt{\Delta}}\right),$$

$$v_{23,24} = a_0$$
$$\mp 3i\sqrt{2}\left(\frac{(\pm 486i\sqrt{2}\lambda^2 l^2 + 648\lambda^2 l^2 \pm 45i\sqrt{2} - 8)\sinh\left(\frac{\sqrt{\Delta}}{4}\lambda\sigma\right)\cosh\left(\frac{\sqrt{\Delta}}{4}\lambda\sigma\right)}{162\lambda(\pm 3i\sqrt{2}+4)\left(2l\sinh\left(\frac{\sqrt{\Delta}}{4}\lambda\sigma\right)\cosh\left(\frac{\sqrt{\Delta}}{4}\lambda\sigma\right) - 2\sqrt{\Delta}\cosh^2\left(\frac{\sqrt{\Delta}}{4}\lambda\sigma\right)\right) + \sqrt{\Delta}}\right),$$

Family3.2

When$\Delta = l^2 - 4mn < 0$ and $lm \neq 0$ or $mn \neq 0$. In Eq. (10), we compensate for the values of a_0 , a_1 and Q_{13} in family 2.2, the solutions of Eq. (6) are given by:

$$v_{25,26} = a_0 \pm 3i\sqrt{2}m\lambda\left(\frac{1}{2m}\left(-l + \sqrt{-\Delta}\left(\tan\left(\frac{\sqrt{-\Delta}}{2}\lambda\sigma\right)\right)\right)\right),$$

simplifying we get

$$v_{25,26} = a_0 \pm \frac{3i\sqrt{2}\lambda}{2}\left(-l + \sqrt{-\Delta}\left(\tan\left(\frac{\sqrt{-\Delta}}{2}\lambda\sigma\right)\right)\right),$$

similarly, we find other solutions,

$$v_{27,28} = a_0 \mp \frac{3i\sqrt{2}\lambda}{2}\left(l + \sqrt{-\Delta}\left(\cot\left(\frac{\sqrt{-\Delta}}{2}\lambda\sigma\right)\right)\right),$$

$$v_{29,30} = a_0 \pm \frac{3i\sqrt{2}\lambda}{2}\left(-l + \sqrt{-\Delta}\left(\tan(\sqrt{-\Delta}\lambda\sigma) \pm \sec(\sqrt{-\Delta}\lambda\sigma)\right)\right),$$

$$v_{31,32} = a_0 \mp \frac{3i\sqrt{2}\lambda}{2}\left(l + \sqrt{-\Delta}\left(\cot(\sqrt{-\Delta}\lambda\sigma) \pm \csc(\sqrt{-\Delta}\lambda\sigma)\right)\right),$$

$$v_{33,34} = a_0 \pm \frac{3i\sqrt{2}\lambda}{4}\left(-2l + \sqrt{-\Delta}\left(\tan\left(\frac{\sqrt{-\Delta}}{4}\lambda\sigma\right) - \cot\left(\frac{\sqrt{-\Delta}}{4}\lambda\sigma\right)\right)\right),$$

$$v_{35,36}=a_0\pm\frac{3i\sqrt{2}\lambda}{2}\left(-l+\frac{\sqrt{-\Delta(\mathrm{M}^2-\mathrm{N}^2)}-\mathrm{M}\sqrt{-\Delta}\cos\left(\sqrt{-\Delta}\lambda\sigma\right)}{\mathrm{M}\sin\left(\sqrt{-\Delta}\lambda\sigma\right)+N}\right),$$

$$v_{37,38}=a_0\pm\frac{3i\sqrt{2}\lambda}{2}\left(-l-\frac{\sqrt{\Delta(\mathrm{M}^2-\mathrm{N}^2)}+\mathrm{M}\sqrt{-\Delta}\cos\left(\sqrt{-\Delta}\lambda\sigma\right)}{\mathrm{M}\sin\left(\sqrt{-\Delta}\lambda\sigma\right)+N}\right),$$

$$v_{39,40}=a_0\mp3i\sqrt{2}\left(\frac{\left(\pm486i\sqrt{2}\lambda^2l^2+648\lambda^2l^2\pm45i\sqrt{2}-8\right)\cos\left(\frac{\sqrt{-\Delta}}{2}\lambda\sigma\right)}{324\lambda(\pm3i\sqrt{2}+4)\left(\sqrt{-\Delta}\sin\left(\frac{\sqrt{\Delta}}{2}\lambda\sigma\right)+l\cos\left(\frac{\sqrt{-\Delta}}{2}\lambda\sigma\right)\right)}\right),$$

$$v_{41,42}=a_0\mp3i\sqrt{2}\left(\frac{\left(\pm486i\sqrt{2}\lambda^2l^2+648\lambda^2l^2\pm45i\sqrt{2}-8\right)\sin\left(\frac{\sqrt{-\Delta}}{2}\lambda\sigma\right)}{324\lambda\left(\pm3i\sqrt{2}+4\right)\left(l\sin\left(\frac{\sqrt{-\Delta}}{2}\lambda\sigma\right)-\sqrt{-\Delta}\cos\left(\frac{\sqrt{-\Delta}}{2}\lambda\sigma\right)\right)}\right),$$

$$v_{43,44}=a_0\mp3i\sqrt{2}\left(\frac{\left(\pm486i\sqrt{2}\lambda^2l^2+648\lambda^2l^2\pm45i\sqrt{2}-8\right)\cos\left(\sqrt{-\Delta}\lambda\sigma\right)}{324\lambda\left(\pm3i\sqrt{2}+4\right)\left(\left(\sqrt{-\Delta}\sin\left(\sqrt{-\Delta}\lambda\sigma\right)+l\cos\left(\sqrt{-\Delta}\lambda\sigma\right)\right)+\sqrt{-\Delta}\right)}\right),$$

$$v_{45,46}=a_0\mp3i\sqrt{2}\left(\frac{\left(\pm486i\sqrt{2}\lambda^2l^2+648\lambda^2l^2\pm45i\sqrt{2}-8\right)\sin\left(\sqrt{-\Delta}\lambda\sigma\right)}{324\lambda\left(\pm3i\sqrt{2}+4\right)\left(\left(l\sin\left(\sqrt{-\Delta}\lambda\sigma\right)-\sqrt{-\Delta}\cos\left(\sqrt{-\Delta}\lambda\sigma\right)\right)+\sqrt{-\Delta}\right)}\right),$$

$$v_{47,48}=a_0\mp3i\sqrt{2}\left(\frac{\left(\pm486i\sqrt{2}\lambda^2l^2+648\lambda^2l^2\pm45i\sqrt{2}-8\right)\sin\left(\frac{\sqrt{-\Delta}}{4}\lambda\sigma\right)\cos\left(\frac{\sqrt{-\Delta}}{4}\lambda\sigma\right)}{162\lambda\left(\pm3i\sqrt{2}+4\right)\left(2l\sin\left(\frac{\sqrt{-\Delta}}{4}\lambda\sigma\right)\cos\left(\frac{\sqrt{-\Delta}}{4}\lambda\sigma\right)-2\sqrt{-\Delta}\cos^2\left(\frac{\sqrt{-\Delta}}{4}\lambda\sigma\right)+\sqrt{-\Delta}\right)}\right),$$

where $\Delta=l^2-\frac{\pm486i\sqrt{2}\lambda^2l^2+648\lambda^2l^2\pm45i\sqrt{2}-8}{162(\lambda^2(\pm3i\sqrt{2}+4)}$

$$\sigma=x+y+z-\left(\frac{\pm135i\sqrt{2}+146}{\pm54\left(3i\sqrt{2}+4\right)}\right)t,\ a_0=\pm\frac{i\left(18l\lambda\pm3i\sqrt{2}-2\right)\sqrt{2}}{12}.$$

The Potential (1+3)-Zakharov-Kuznetsov EquationApplication

In this section, we present our proposed equation, namelypotential the (3+1)-dimensional Zakharov-Kuznetsov (pZK) equation as the form:

$$u_t+a(u_x)u_x+b(u_{xx}+u_{yy}+\mathrm{u_{zz}})_x=0, \qquad (11)$$

and donated by (pZK),

where

$$u_t+au_x+b(u_{xx}+u_{yy}+\mathrm{u_{zz}})_x=0, \qquad (12)$$

is the (3+1)-dimensional Zakharov-Kuznetsov (ZK).

Now, we apply the improvedgeneralized Riccati equation mapping method to find many families of exact traveling wave solutions of Eq. (11). That will be transformed to the ODE

$$-cu'+\mathrm{a}\lambda(\mathrm{u}')^2+3b\lambda^2u'''=0. \qquad (13)$$

By using the wave variable $\mu=\lambda(x+y-ct)$.

Balancing thehighest order of the nonlinear term $(\mathrm{u}')^2$ with the highest order derivative u''', we get $m+3=2(m+1)$, that gives $m=1$.

Hence the formal solution of Eq.(13) takes the form:

$$u(\mu)=a_0+a_1Q, \qquad (14)$$

where a_0 and a_1 are constants to be determined.

Substituting Eq. (14) in to Eq. (13), collecting the coefficients of Q and solving the resulting system using maple program,we obtain the following one solution:

$a_0 = 0, a_1 = \dfrac{-18bm\lambda}{a}, c = 3bl^2\lambda^2 - 12bmn\lambda^2, \lambda = \lambda, m = m, l = l, n = n$

Using Eq. (13), the solutions of Eq. (14),

Family 4.1:

When$\Delta = l^2 - 4mn > 0$ and $lm \neq 0$ or $mn \neq 0$. In Eq. (14), we compensate for the values of a_0, a_1 and Q_1 in family 2.1, the solutions of Eq. (13) are given by:

$$u_1 = \frac{-18bm\lambda}{a}\left(\frac{-1}{2m}\left(l + \sqrt{\Delta}\left(\tanh\left(\frac{\sqrt{\Delta}}{2}\lambda\varphi\right)\right)\right)\right),$$

Simplifying, we get

$$u_1 = \frac{9b\lambda}{\text{a}}\left(l + \sqrt{\Delta}\left(\tanh\left(\frac{\sqrt{\Delta}}{2}\lambda\varphi\right)\right)\right),$$

similarly, we find other solutions,

$$u_2 = \frac{9b\lambda}{\text{a}}\left(l + \sqrt{\Delta}\left(\coth\left(\frac{\sqrt{\Delta}}{2}\lambda\varphi\right)\right)\right),$$

$$u_3 = \frac{9b\lambda}{\text{a}}\left(l + \sqrt{\Delta}\left(\tanh(\sqrt{\Delta}\lambda\varphi) + i\text{sech}(\sqrt{\Delta}\lambda\varphi)\right)\right),$$

$$u_4 = \frac{9b\lambda}{\text{a}}\left(l + \sqrt{\Delta}\left(\coth(\sqrt{\Delta}\lambda\varphi) + cs\text{ch}(\sqrt{\Delta}\lambda\varphi)\right)\right),$$

$$u_5 = \frac{9b\lambda}{\text{a}}\left(l + \frac{\sqrt{\Delta}}{2}\left(\tanh\left(\frac{\sqrt{\Delta}}{4}\lambda\varphi\right) + cot\text{h}\left(\frac{\sqrt{\Delta}}{4}\lambda\varphi\right)\right)\right),$$

$$u_6 = \frac{9b\lambda}{\text{a}}\left(l + \frac{\sqrt{\Delta(M^2 + N^2)} + M\sqrt{\Delta}(\cosh(\sqrt{\Delta}\lambda\varphi)}{\text{A}\sinh(\sqrt{\Delta}\lambda\varphi) + N}\right),$$

$$u_7 = \frac{9b\lambda}{\text{a}}\left(l - \frac{\sqrt{\Delta(N^2 + M^2)} - M\sqrt{\Delta}\left(\sinh(\sqrt{\Delta}\lambda\varphi)\right)}{\text{A}\cosh(\sqrt{\Delta}\lambda\varphi) + N}\right),$$

$$u_8 = \frac{-36bmn\lambda\cosh\left(\frac{\sqrt{\Delta}}{2}\lambda\varphi\right)}{\text{a}\left(\sqrt{\Delta}\sinh\left(\frac{\sqrt{\Delta}}{2}\lambda\varphi\right) - l\cosh\left(\frac{\sqrt{\Delta}}{2}\lambda\varphi\right)\right)},$$

$$u_9 = \frac{36bmn\lambda\sinh\left(\frac{\sqrt{\Delta}}{2}\lambda\varphi\right)}{\text{a}\left(l\sinh\left(\frac{\sqrt{\Delta}}{2}\lambda\varphi\right) - \sqrt{\Delta}\cosh\left(\frac{\sqrt{\Delta}}{2}\lambda\varphi\right)\right)},$$

$$u_{10} = \frac{-36bmn\lambda\cosh(\sqrt{\Delta}\lambda\varphi)}{\text{a}\left(\sqrt{\Delta}\sinh(\sqrt{\Delta}\lambda\varphi) - l\cosh(\sqrt{\Delta}\lambda\varphi) \pm i\sqrt{\Delta}\right)},$$

$$u_{11} = \frac{36bmn\lambda\sinh(\sqrt{\Delta}\lambda\varphi)}{\text{a}\left(l\sinh(\sqrt{\Delta}\lambda\varphi) - \sqrt{\Delta}\cosh(\sqrt{\Delta}\lambda\varphi) \pm \sqrt{\Delta}\right)},$$

$$u_{12} = \frac{72bmn\lambda\sinh\left(\frac{\sqrt{\Delta}}{4}\lambda\varphi\right)\cosh\left(\frac{\sqrt{\Delta}}{4}\lambda\varphi\right)}{\text{a}\left(2l\sinh\left(\frac{\sqrt{\Delta}}{4}\lambda\varphi\right)\cosh\left(\frac{\sqrt{\Delta}}{4}\lambda\varphi\right) - 2\sqrt{\Delta}\cosh\left(\frac{\sqrt{\Delta}}{4}\lambda\varphi\right) + \sqrt{\Delta}\right)},$$

Family 4.2:

When$\Delta = l^2 - 4mn < 0$ and $lm \neq 0$ or $mn \neq 0$. In Eq. (14), we compensate for the values of a_0, a_1 and Q_{13} in family 2.2, the solutions of Eq. (13) are given by:

$$u_{13} = \frac{-18bm\lambda}{a}\left(\frac{1}{2m}\left(-l + \sqrt{\Delta}\left(\tan\left(\frac{\sqrt{\Delta}}{2}\lambda\varphi\right)\right)\right)\right),$$

simplifying we get

$$u_{13} = \frac{9b\lambda}{\mathrm{a}}\left(l - \sqrt{-\Delta}\left(\tan\left(\frac{\sqrt{-\Delta}}{2}\lambda\varphi\right)\right)\right),$$

similarly, we find other solutions,

$$u_{14} = \frac{9b\lambda}{\mathrm{a}}\left(l + \sqrt{-\Delta}\left(\cot\left(\frac{\sqrt{-\Delta}}{2}\lambda\varphi\right)\right)\right),$$

$$u_{15} = \frac{9b\lambda}{\mathrm{a}}\left(l - \sqrt{-\Delta}\left(\tan\left(\sqrt{-\Delta}\lambda\varphi\right) + \sec\left(\sqrt{-\Delta}\lambda\varphi\right)\right)\right),$$

$$u_{16} = \frac{9b\lambda}{\mathrm{a}}\left(l + \sqrt{-\Delta}\left(\cot\left(\sqrt{-\Delta}\lambda\varphi\right) + csc\left(\sqrt{-\Delta}\lambda\varphi\right)\right)\right),$$

$$u_{17} = \frac{9b\lambda}{2\mathrm{a}}(2l - \sqrt{-\Delta}\left(\tan\left(\frac{\sqrt{-\Delta}}{4}\lambda\varphi\right) - cot\left(\frac{\sqrt{-\Delta}}{4}\lambda\varphi\right)\right),$$

$$u_{18} = \frac{9b\lambda}{\mathrm{a}}\left(l + \frac{\sqrt{-\Delta(M^2 - N^2)} + M\sqrt{-\Delta}(\cos\left(\sqrt{-\Delta}\lambda\varphi\right)}{\mathrm{A}\sin\left(\sqrt{-\Delta}\lambda\varphi\right) + N}\right),$$

$$u_{19} = \frac{9b\lambda}{\mathrm{a}}(l - \frac{\sqrt{-\Delta(M^2 - N^2)} - M\sqrt{-\Delta}(\cos\left(\sqrt{-\Delta}\lambda\varphi\right)}{\mathrm{A}\sin\left(\sqrt{-\Delta}\lambda\varphi\right) + N},$$

$$u_{20} = \frac{36bmn\lambda\cos\left(\frac{\sqrt{-\Delta}}{2}\lambda\varphi\right)}{\mathrm{a}\left(\sqrt{-\Delta}\sin\left(\frac{\sqrt{-\Delta}}{2}\lambda\varphi\right) + l\cos\left(\frac{\sqrt{-\Delta}}{2}\lambda\varphi\right)\right)},$$

$$u_{21} = \frac{36bmn\lambda\sin\left(\frac{\sqrt{-\Delta}}{2}\lambda\varphi\right)}{\mathrm{a}\left(l\sin\left(\frac{\sqrt{-\Delta}}{2}\lambda\varphi\right) - \sqrt{\Delta}\cos\left(\frac{\sqrt{-\Delta}}{2}\lambda\varphi\right)\right)},$$

$$u_{22} = \frac{36bmn\lambda\cos\left(\sqrt{-\Delta}\lambda\varphi\right)}{\mathrm{a}\left(\sqrt{-\Delta}\sin\left(\sqrt{-\Delta}\lambda\varphi\right) + l\cos\left(\sqrt{-\Delta}\lambda\varphi\right) \pm \sqrt{\Delta}\right)},$$

$$u_{23} = \frac{36bmn\lambda\sin\left(\sqrt{-\Delta}\lambda\varphi\right)}{\mathrm{a}\left(l\sin\left(\sqrt{-\Delta}\lambda\varphi\right) - \sqrt{-\Delta}\cos\left(\sqrt{-\Delta}\lambda\varphi\right) \pm \sqrt{-\Delta}\right)},$$

$$u_{24} = \frac{72bmn\lambda\sin\left(\frac{\sqrt{-\Delta}}{4}\lambda\varphi\right)\cos\left(\frac{\sqrt{-\Delta}}{4}\lambda\varphi\right)}{\mathrm{a}\left(2l\sin\left(\frac{\sqrt{-\Delta}}{4}\lambda\varphi\right)\cos\left(\frac{\sqrt{-\Delta}}{4}\lambda\varphi\right) - 2\sqrt{-\Delta}\cos\left(\frac{\sqrt{-\Delta}}{4}\lambda\varphi\right) + \sqrt{-\Delta}\right)},$$

where$\Delta = l^2 - 4mn, \varphi = x + y + z - (3bl^2\lambda^2 - 12bmn\lambda^2)$.

Conclusion

In summary, the improved Riccati equation method has been proposed and used to find out exact solutions of nonlinear equation with aid maple.

Our method allows us to carry out the solution process of nonlinear wave equations more systematically and conveniently by computer algebra systems such as Maple.We have successfully obtained some travelling wave solutions of the (cmZKB)equation and a potential of(ZK). When the parameters aretaken as special values, the solitary wave solutions and periodic wave solutions are obtained. We surely believe that these solutions will be of great importance for analyzing the nonlinear phenomena arising in applied physical sciences. The work shows international journal of differential equations that the improved Riccati equation method is sufficient, effective and suitable for solving other nonlinear evolution equations and it deserves further applying and studying, as well.

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طريقة معادلة ريكاتي المعممة لحل المعادلات (pZK) و(cmZKB)

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الملخص

في هذا البحث قمنا بتطبيق معادلةريكاتيالمعممة لإيجاد حلول معادلةزاخروف–كازنيستوف-بيرغر ومعادلة البوتينشلللزخروف – كازانيستوف. إذ حصلنا على العديد من العائلات الجديدة من حلول موجة السفر الجديدة الدقيقة والمعبرة عنها بواسطة الدوال المثلثية، الدوال الزائدية، الدوال الكسرية. إذ أنَّتطبيق معادلة ريكاتي المعممة تعد أداة قوية لحل العديد من المعادلات التفاضليةالخطية في الرياضيات وفي العلوم الفيزيائية.

الكلمات المفتاحية: تطبيق معادلة ريكاتي المعممة، الحلول الدقيقة، حلول معادلة زاخروف – كازنيستوف-بيرغر(3+1) ومعادلة البوتينشلللزخروف – كازانيستوف (3+1).