An application of N-Fractional calculus to solve ordinary differential equation of fourth order

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Abstract

There are many papers which have been published in the same direction of this paper by K. Nishimoto, S. Owa, Shih-Tong Tu and H. M. Srivastava [7],[8]. In this paper, application of N-fractional calculus to solve ordinary differential equation of fourth order is obtained.

Key words: N-fractional calculus, ordinary differential equation, homogeneous and non-homogeneous differential equations.

Introduction:

K. Nishimoto defined the fractional calculus of order (ν) for functions of single variable as the following:

Definition: (Nishimoto K. [4], [5], [6]):
Let $D=\{D_-, D_+\}$, $C=\{ C_-, C_+\}$, where
C_- be a curve along the cut joining two points z and $-\infty + i\,\mathrm{Im}(z)$,
C_+ be a curve along the cut joining two points z and $\infty + i\,\mathrm{Im}(z)$,
D_- be a domain surrounded by C_- ,
D_+ be a domain surrounded by C_+ .
(Here D contains the points over curves C).
Moreover , let $f = f(z)$ be a regular function in D (z∈D) ,

$$f_\nu = {}_C(f)_\nu(z) = \frac{\Gamma(\nu+1)}{2\pi i}\int_C \frac{f(\varsigma)}{(\varsigma - z)^{\nu+1}}\,d\varsigma \qquad (\nu \notin z^-)$$

$$(f)_{-m} = \lim_{\nu \to -m} (f)_\nu \qquad (m \in z^+) ,$$

where $\varsigma \neq z$, $z \in C$, $\nu \in R$, Γ :Gamma function, $-\pi \leq \arg(\varsigma - z) \leq \pi$ for C_-,
$0 \leq \arg(\varsigma - z) \leq 2\pi$ for C_+,
then $(f)_\nu$ is the fractional differintegration of arbitrary order ν (derivatives of order ν for $\nu > 0$, and integrals of order $-\nu$ for $\nu < 0$) , with respect to z, of the function f , if $|(f)_\nu|$ $. < \infty$

Lemmas

In order to discuss the solution of ordinary differential equation of fourth order,

we need the following lemmas [2].

Lemma (1): If k is constant, then
$(k)_\nu = k \cdot (1)_\nu = 0$, for $\nu \notin Z^- \cup \{0\}$

Lemma (2): Let $f = f(z)$ be regular function, if f_ν exist, then

$$(fa)_\nu = \frac{\Gamma(\nu+1)}{2\pi i}\int_c \frac{f(\zeta)\cdot a}{(\zeta - z)^{\nu+1}}\, d\zeta$$

$(fa)_\nu = af_\nu$, where a is constant $(z,\nu \in C)$.

Lemma (3): Let $u = u(z)$, $v = v(z)$ be regular functions and if u_ν, v_ν exist, then $(ua + vb)_\nu = a(u)_\nu + b(v)_\nu$, where a,b are constants $(z,\nu \in C)$.

Lemma (4): Let $u = u(z)$, $v = v(z)$ be regular functions and if are u_ν, v_ν exist, then

$$(u\,v)_\nu = \sum_{n=0}^{\infty} \frac{\Gamma(\nu+1)}{\Gamma(\nu-n+1)\Gamma(n+1)}(v)_{\nu-n}(u)_n, \qquad (z,\nu \in C).$$

Solution of Non-homogeneous Ordinary Differential Equation

Theorem:

If $f_{-\alpha}$ is exist and $f_{-\alpha} \neq 0$, then the differential equation

$$\begin{aligned}
&\varphi_4\left[a^2bZ^4 + a^2Z^3 - b\beta^2Z^2 - \beta^2 Z\right] + \\
&+\varphi_3\left\{ abZ^3\left(4a\alpha + \beta\gamma + a\right) + Z^2\left(3a^2\alpha + a\beta\gamma + b\beta\cdot[\beta^2 + \beta\gamma + \frac{\delta}{\beta}]\right) + \right.\\
&\left. + Z\beta\left((\beta^2 + \beta\gamma + \frac{\delta}{\beta}) - b\beta\cdot[2\alpha+1]\right) - \alpha\beta^2\right\} + \\
&+\alpha\varphi_2\left\{abZ^2\left(12a\cdot[\alpha-1] + 3\cdot[\beta\gamma + a]\right) + + Z\left(6a^2[\alpha-1] + 2a\beta\gamma + 2b\beta\cdot[\beta^2 + \beta\gamma + \frac{\delta}{\beta}]\right) + \right.\\
&\left.\left(\beta(\beta^2 + \beta\gamma + \frac{\delta}{\beta}) - b\beta^2[2\alpha-1]\right)\right\} + \\
&+\alpha(\alpha-1)\varphi_1\left\{6abZ\left(4a[\alpha-2] + \beta\gamma + a\right) + + \left(6a^2[\alpha-2] + 2a\beta\gamma + 2b\beta\cdot[\beta^2 + \beta\gamma + \frac{\delta}{\beta}]\right)\right\} + \\
&+\alpha(\alpha-1)(\alpha-2)\varphi\left\{6ab\left(4a(\alpha-3) + \beta\gamma + a\right)\right\} = f\,,\ (\ a\beta \neq 0 \text{ and } Z \neq 0\ ) \qquad (1)
\end{aligned}$$

has a particular solution of the form

$$\varphi = \left(\left(f_{-\alpha}\,\frac{P(Z)}{Z(aZ-\beta)(aZ+\beta)(bZ+1)}\right)_{-1}\cdot\left(P(Z)\right)^{-1}\right)_{\alpha-3}, \qquad (2)$$

for arbitrary α ,where $\varphi = \varphi(z)$, $z \in C$, $\varphi_m = \dfrac{d^m \varphi}{dz^m}$, $m = 0,1,2,3,4$, $f = f(z)$ is known,

$$P(Z) = \frac{(aZ-\beta)^{\frac{C}{a}} \cdot (bZ+1)}{(aZ+\beta)^{\frac{E}{a}}}, \quad \text{where } C = \tfrac{1}{2}(\beta^2 + 2\beta\gamma + \frac{\delta}{\beta}) \text{ and } E = \tfrac{1}{2}(\beta^2 + \frac{\delta}{\beta}),$$

and a, b, β, γ and δ are constants.

Proof:

By applying the operational properties of the generalized operator to

$$\varphi = w_\alpha \qquad \text{(see [1], [3])} \qquad (3)$$

then

$$\varphi_1 = w_{\alpha+1} \qquad (4)$$

$$\varphi_2 = w_{\alpha+2} \qquad (5)$$

$$\varphi_3 = w_{\alpha+3} \qquad (6)$$

and

$$\varphi_4 = w_{\alpha+4} \qquad (7)$$

where $\varphi_k = \dfrac{d^k \varphi}{dz^k}$, $k = 0,1,2,3,4$ and $w = w(z)$.

Substituting (3) - (7) into (1); we get

$$w_{\alpha+4}\left[a^2bZ^4 + a^2Z^3 - b\beta^2Z^2 - \beta^2 Z\right] +$$

$$+ w_{\alpha+3}\left\{Z^3ab(4a\alpha + \beta\gamma + a) + Z^2\left(3a^2\alpha + a\beta\gamma + b\beta(\beta^2 + \beta\gamma + \frac{\delta}{\beta})\right)\right.$$

$$\left. + Z\beta\left((\beta^2 + \beta\gamma + \frac{\delta}{\beta}) - b\beta(2\alpha+1)\right) - \alpha\beta^2\right\} +$$

$$+ \alpha \cdot w_{\alpha+2}\left\{Z^2ab\left(12a(\alpha-1) + 3(\beta\gamma + a)\right) + Z\left(6a^2(\alpha-1) + 2a\beta\gamma + 2b\beta(\beta^2 + \beta\gamma + \frac{\delta}{\beta})\right) +\right.$$

$$\left. + \left(\beta(\beta^2 + \beta\gamma + \frac{\delta}{\beta}) - b\beta^2[2\alpha - 1]\right)\right\} +$$

$$+ \alpha(\alpha-1)w_{\alpha+1}\left\{6abZ\left(4a(\alpha-2) + \beta\gamma + a\right) + \left(6a^2(\alpha-2) + 2a\beta\gamma + 2b\beta(\beta^2 + \beta\gamma + \frac{\delta}{\beta})\right)\right\} +$$

$$+ \alpha(\alpha-1)(\alpha-2)w_\alpha\left\{6ab\left(4a(\alpha-3) + \beta\gamma + a\right)\right\} = f$$

or

$$w_{\alpha+4}\left\{a^2bZ^4+a^2Z^3-b\beta^2Z^2-\beta^2Z\right\}+w_{\alpha+3}\left\{\alpha[4a^2bZ^3+3a^2Z^2-2b\beta^2Z-\beta^2]+\right.$$

$$\left.+Z^3[ab\beta\gamma+a^2b]+Z^2[a\beta\gamma+b\beta(\beta^2+\frac{\delta}{\beta})+b\beta^2\gamma]+Z[\beta(\beta^2+\frac{\delta}{\beta})+\beta^2\gamma-b\beta^2]\right\}+$$

$$+w_{\alpha+2}\cdot\alpha\left\{(\alpha-1)[12a^2bZ^2+6a^2Z-2b\beta^2]+3Z^2[ab\beta\gamma+a^2b]+2Z[a\beta\gamma+b\beta(\beta^2+\frac{\delta}{\beta})+b\beta^2\gamma]+\right.$$

$$\left.+[\beta(\beta^2+\frac{\delta}{\beta})+\beta^2\gamma-b\beta^2]\right\}+w_{\alpha+1}\alpha(\alpha-1)\left\{(\alpha-2)[24a^2bZ+6a^2]+6Z[ab\beta\gamma+a^2b]+\right.$$

$$\left.+2\cdot[a\beta\gamma+b\beta(\beta^2+\frac{\delta}{\beta})+b\beta^2\gamma]\right\}+w_\alpha\alpha(\alpha-1)(\alpha-2)\left\{24a^2b(\alpha-3)+6[ab\beta\gamma+a^2b]\right\}=f$$

and

$$\left[w_{\alpha+4}\left\{a^2bZ^4+a^2Z^3-b\beta^2Z^2-\beta^2Z\right\}+\alpha w_{\alpha+3}\left\{4a^2bZ^3+3a^2Z^2-2b\beta^2Z-\beta^2\right\}+\right.$$

$$+\alpha(\alpha-1)w_{\alpha+2}\left\{12a^2bZ^2+6a^2Z-2b\beta^2\right\}+\alpha(\alpha-1)(\alpha-2)w_{\alpha+1}\left\{24a^2bZ+6a^2\right\}+$$

$$\left.+\alpha(\alpha-1)(\alpha-2)(\alpha-3)w_\alpha\left\{24a^2b\right\}\right]+$$

$$+\left[w_{\alpha+3}\left\{Z^3[ab\beta\gamma+a^2b]+Z^2[a\beta\gamma+b\beta(\beta^2+\frac{\delta}{\beta})+b\beta^2\gamma]+Z[\beta(\beta^2+\frac{\delta}{\beta})+\beta^2\gamma-b\beta^2]\right\}+\right.$$

$$+\alpha\cdot w_{\alpha+2}\left\{3Z^2[ab\beta\gamma+a^2b]+2Z[a\beta\gamma+b\beta(\beta^2+\frac{\delta}{\beta})+b\beta^2\gamma]+[\beta(\beta^2+\frac{\delta}{\beta})+\beta^2\gamma-b\beta^2]\right\}+$$

$$+\alpha(\alpha-1)w_{\alpha+1}\left\{6Z[ab\beta\gamma+a^2b]+2\cdot[a\beta\gamma+b\beta(\beta^2+\frac{\delta}{\beta})+b\beta^2\gamma]\right\}+$$

$$\left.+\alpha(\alpha-1)(\alpha-2)w_\alpha\left\{6\cdot[ab\beta\gamma+a^2b]\right\}\right]=f\,, \qquad (8)$$

Now, by employing lemma (4), equation (8) reduces to

$$\left(w_4\left\{a^2bZ^4+a^2Z^3-b\beta^2Z^2-\beta^2Z\right\}\right)_\alpha+$$

$$+\left(w_3\left\{Z^3[ab\beta\gamma+a^2b]+Z^2[a\beta\gamma+b\beta(\beta^2+\frac{\delta}{\beta})+b\beta^2\gamma]+Z[\beta(\beta^2+\frac{\delta}{\beta})+\beta^2\gamma-b\beta^2]\right\}\right)_\alpha=f \qquad (9)$$

By using lemma (3) then taking the differintegration of order $(-\alpha)$ of (9), we get

$$w_4\left\{a^2bZ^4+a^2Z^3-b\beta^2Z^2-\beta^2Z\right\}+$$

$$+w_3\left\{Z^3\left[\tfrac{1}{2}ab(\beta^2+2\beta\gamma+\frac{\delta}{\beta})-\tfrac{1}{2}ab(\beta^2+\frac{\delta}{\beta})+a^2b\right]+\right.$$

$$+Z^2\left[\tfrac{1}{2}a(\beta^2+2\beta\gamma+\frac{\delta}{\beta})+\tfrac{1}{2}b\beta(\beta^2+2\beta\gamma+\frac{\delta}{\beta})-\tfrac{1}{2}a(\beta^2+\frac{\delta}{\beta})+\tfrac{1}{2}b\beta(\beta^2+\frac{\delta}{\beta})\right]+$$

$$\left.+Z\left[\tfrac{1}{2}\beta(\beta^2+2\beta\gamma+\frac{\delta}{\beta})+\tfrac{1}{2}\beta(\beta^2+\frac{\delta}{\beta})-b\beta^2\right]\right\}=f_{-\alpha} \qquad (10)$$

Dividing (10) by $\left(a^2bZ^4+a^2Z^3-b\beta^2Z^2-\beta^2Z\right)$, and let $C=\tfrac{1}{2}(\beta^2+2\beta\gamma+\frac{\delta}{\beta})$ and $E=\tfrac{1}{2}(\beta^2+\frac{\delta}{\beta})$; we obtain

$$w_4+w_3\,\frac{Z^3ab[C-E+a]+Z^2[a(C-E)+b\beta(C+E)]+Z\beta[C+E-b\beta]}{a^2bZ^4+a^2Z^3-b\beta^2Z^2-\beta^2Z}=$$

$$f_{-\alpha}\cdot\frac{1}{a^2bZ^4+a^2Z^3-b\beta^2Z^2-\beta^2Z} \qquad (11)$$

Multiplying (11) by $P(Z)$, where $P(Z)=\dfrac{(aZ-\beta)^{\frac{C}{a}}\cdot(bZ+1)}{(aZ+\beta)^{\frac{E}{a}}}$; we obtain

$$w_4\cdot P(Z)+w_3\cdot\left(P(Z)\right)_1=f_{-\alpha}\,\frac{P(Z)}{a^2bZ^4+a^2Z^3-b\beta^2Z^2-\beta^2Z}$$

i.e,

$$\left(w_3\cdot P(Z)\right)_1=f_{-\alpha}\,\frac{P(Z)}{Z(aZ-\beta)(aZ+\beta)(bZ+1)}$$

and

$$w_3\cdot P(Z)=\left(f_{-\alpha}\,\frac{P(Z)}{Z(aZ-\beta)(aZ+\beta)(bZ+1)}\right)_{-1}$$

or

$$w_3=\left(f_{-\alpha}\,\frac{P(Z)}{Z(aZ-\beta)(aZ+\beta)(bZ+1)}\right)_{-1}\cdot\left(P(Z)\right)^{-1}$$

that is

$$w = \left(\left(f_{-\alpha} \frac{P(Z)}{Z(aZ-\beta)(aZ+\beta)(bZ+1)} \right)_{-1} \cdot (P(Z))^{-1} \right)_{-3} \quad (12)$$

From (3) and (12); we get

$$\varphi = w_{\alpha} = \left(\left(f_{-\alpha} \frac{P(Z)}{Z(aZ-\beta)(aZ+\beta)(bZ+1)} \right)_{-1} \cdot (P(Z))^{-1} \right)_{\alpha-3},$$

as a particular solution to the differential equation (1).

Verification of The Solution:

Routing inversely, we have

$$\varphi = w_{\alpha} = \left(\left(f_{-\alpha} \frac{P(Z)}{Z(aZ-\beta)(aZ+\beta)(bZ+1)} \right)_{-1} \cdot (P(Z))^{-1} \right)_{\alpha-3} \quad (13)$$

then

$$\varphi_1 = w_{\alpha+1} = \left(\left(f_{-\alpha} \frac{P(Z)}{Z(aZ-\beta)(aZ+\beta)(bZ+1)} \right)_{-1} \cdot (P(Z))^{-1} \right)_{\alpha-2}, \quad (14)$$

$$\varphi_2 = w_{\alpha+2} = \left(\left(f_{-\alpha} \frac{P(Z)}{Z(aZ-\beta)(aZ+\beta)(bZ+1)} \right)_{-1} \cdot (P(Z))^{-1} \right)_{\alpha-1}, \quad (15)$$

$$\varphi_3 = w_{\alpha+3} = \left(\left(f_{-\alpha} \frac{P(Z)}{Z(aZ-\beta)(aZ+\beta)(bZ+1)} \right)_{-1} \cdot (P(Z))^{-1} \right)_{\alpha}, \quad (16)$$

and

$$\varphi_4 = w_{\alpha+4} = \left(\left(f_{-\alpha} \frac{P(Z)}{Z(aZ-\beta)(aZ+\beta)(bZ+1)} \right)_{-1} \cdot (P(Z))^{-1} \right)_{\alpha+1}. \quad (17)$$

Substituting (13) - (17) into the left – hand side of (1), we get

$$\text{L. H. S. of } (1) = \varphi_4 \left[a^2bZ^4 + a^2Z^3 - b\beta^2Z^2 - \beta^2 Z \right] +$$

$$+\varphi_3 \left\{ Z^3 ab(4a\alpha + \beta\gamma + a) + Z^2 \left(3a^2\alpha + a\beta\gamma + b\beta(\beta^2 + \beta\gamma + \frac{\delta}{\beta}) \right) + \right.$$

$$\left. + Z\beta \left((\beta^2 + \beta\gamma + \frac{\delta}{\beta}) - b\beta(2\alpha+1) \right) - \alpha\beta^2 \right\} +$$

$$\alpha\varphi_2 \left\{ Z^2 ab\big(12a(\alpha-1) + 3(\beta\gamma + a)\big) + Z\left(6a^2(\alpha-1) + 2a\beta\gamma + 2b\beta(\beta^2 + \beta\gamma + \frac{\delta}{\beta}) \right) + \right.$$

$$\beta(\beta^2+\beta\gamma+\frac{\delta}{\beta})-b\beta^2(2\alpha-1)\Big\}+$$

$$\alpha(\alpha-1)\varphi_1\Big\{6abZ\big(4a(\alpha-2)+\beta\gamma+a\big)+\Big(6a^2(\alpha-2)+2a\beta\gamma+2b\beta(\beta^2+\beta\gamma+\frac{\delta}{\beta})\Big)\Big\}+$$

$$\alpha(\alpha-1)(\alpha-2)\varphi\Big\{6ab\Big(4a(\alpha-3)+\beta\gamma+a\Big)\Big\}=$$

$$w_{\alpha+4}\Big[a^2bZ^4+a^2Z^3-b\beta^2Z^2-\beta^2Z\Big]+$$

$$+w_{\alpha+3}\Big\{Z^3ab(4a\alpha+\beta\gamma+a)+Z^2\Big(3a^2\alpha+a\beta\gamma+b\beta(\beta^2+\beta\gamma+\frac{\delta}{\beta})\Big)+$$

$$+Z\beta\Big((\beta^2+\beta\gamma+\frac{\delta}{\beta})-b\beta(2\alpha+1)\Big)-\alpha\beta^2\Big\}+$$

$$+\alpha\cdot w_{\alpha+2}\Big\{Z^2ab\big(12a(\alpha-1)+3(\beta\gamma+a)\big)+Z\Big(6a^2(\alpha-1)+2a\beta\gamma+2b\beta(\beta^2+\beta\gamma+\frac{\delta}{\beta})\Big)+$$

$$+\beta(\beta^2+\beta\gamma+\frac{\delta}{\beta})-b\beta^2(2\alpha-1)\Big\}+$$

$$+\alpha(\alpha-1)w_{\alpha+1}\Big\{6abZ\big(4a(\alpha-2)+\beta\gamma+a\big)+\Big(6a^2(\alpha-2)+2a\beta\gamma+2b\beta(\beta^2+\beta\gamma+\frac{\delta}{\beta})\Big)\Big\}+$$

$$+\alpha(\alpha-1)(\alpha-2)w_\alpha\Big\{6ab\Big(4a(\alpha-3)+\beta\gamma+a\Big)\Big\}$$

L. H. S. of (1) = $\Big[\ w_{\alpha+4}\Big\{a^2bZ^4+a^2Z^3-b\beta^2Z^2-\beta^2Z\Big\}+$

$$+\alpha\cdot w_{\alpha+3}\Big\{4a^2bZ^3+3a^2Z^2-2b\beta^2Z-\beta^2\Big\}+$$

$$+\alpha(\alpha-1)w_{\alpha+2}\Big\{12a^2bZ^2+6a^2Z-2b\beta^2\Big\}+\alpha(\alpha-1)(\alpha-2)w_{\alpha+1}\Big\{24a^2bZ+6a^2\Big\}+$$

$$+\alpha(\alpha-1)(\alpha-2)(\alpha-3)w_\alpha\Big\{24a^2b\Big\}\Big]+$$

$$+\Big[w_{\alpha+3}\Big\{Z^3[ab\beta\gamma+a^2b]+Z^2[a\beta\gamma+b\beta(\beta^2+\frac{\delta}{\beta})+b\beta^2\gamma]+Z[\beta(\beta^2+\frac{\delta}{\beta})+\beta^2\gamma-b\beta^2]\Big\}+$$

$$+\alpha\cdot w_{\alpha+2}\Big\{3Z^2[ab\beta\gamma+a^2b]+2Z[a\beta\gamma+b\beta(\beta^2+\frac{\delta}{\beta})+b\beta^2\gamma]+[\beta(\beta^2+\frac{\delta}{\beta})+\beta^2\gamma-b\beta^2]\Big\}+$$

$$+\alpha(\alpha-1)w_{\alpha+1}\left\{6Z[ab\beta\gamma+a^2b]+2\cdot[a\beta\gamma+b\beta(\beta^2+\frac{\delta}{\beta})+b\beta^2\gamma]\right\}+$$

$$\left.+\alpha(\alpha-1)(\alpha-2)w_{\alpha}\left\{6\cdot[ab\beta\gamma+a^2b]\quad\right\}\quad\right] \qquad (18)$$

Now, by employing lemma (4), equation (18) reduces to

$$\text{L.H.S. of } (1) = \left(w_4\left\{a^2bZ^4+a^2Z^3-b\beta^2Z^2-\beta^2Z\right\}\right)_{\alpha}+$$

$$+\left(w_3\left\{Z^3[ab\beta\gamma+a^2b]+Z^2[a\beta\gamma+b\beta(\beta^2+\frac{\delta}{\beta})+b\beta^2\gamma]+Z[\beta(\beta^2+\frac{\delta}{\beta})+\beta^2\gamma-b\beta^2]\right\}\right)_{\alpha}$$

(19)

Substituting (9) into (19), we obtain

L.H.S. of (1) = f .

Special Case of the Theorem:

Putting $f=0$ in the theorem; we obtain the homogeneous differential equation

$$\varphi_4\left[a^2bZ^4+a^2Z^3-b\beta^2Z^2-\beta^2Z\right]+$$

$$+\varphi_3\left\{Z^3ab(4a\alpha+\beta\gamma+a)+Z^2\left(3a^2\alpha+a\beta\gamma+b\beta(\beta^2+\beta\gamma+\frac{\delta}{\beta})\right)+\right.$$

$$\left.+Z\beta\left((\beta^2+\beta\gamma+\frac{\delta}{\beta})-b\beta(2\alpha+1)\right)-\alpha\beta^2\right\}+$$

$$+\alpha\varphi_2\left\{Z^2ab\big(12a(\alpha-1)+3(\beta\gamma+a)\big)+Z\left(6a^2(\alpha-1)+2a\beta\gamma+2b\beta(\beta^2+\beta\gamma+\frac{\delta}{\beta})\right)+\right.$$

$$\left.+\beta(\beta^2+\beta\gamma+\frac{\delta}{\beta})-b\beta^2(2\alpha-1)\right\}+$$

$$+\alpha(\alpha-1)\varphi_1\left\{6abZ\big(4a(\alpha-2)+\beta\gamma+a\big)+\left(6a^2(\alpha-2)+2a\beta\gamma+2b\beta(\beta^2+\beta\gamma+\frac{\delta}{\beta})\right)\right\}+$$

$$\alpha(\alpha-1)(\alpha-2)\varphi\left\{6ab\left(4a(\alpha-3)+\beta\gamma+a\right)\right\}=0, \qquad (a\beta\neq0 \text{ and } Z\neq0) \qquad (20)$$

which has a particular solution of the form

$$\varphi=\left(\left((0)_{-\alpha}\cdot\frac{P(Z)}{Z(aZ-\beta)(aZ+\beta)(bZ+1)}\right)_{-1}\cdot\big(P(Z)\big)^{-1}\right)_{\alpha-3}, \qquad (21)$$

By using lemma (1)

$$(0)_{-\alpha} = \left((0)_{-1}\right)_{-\alpha+1} = (m)_{-\alpha+1} = \begin{cases} (m)_{-\alpha+1} = 0, & if \quad (-\alpha+1) \notin Z^- \cup \{0\} \\ (m)_{-\alpha+1} \neq 0, & if \quad (-\alpha+1) \in Z^- \end{cases},$$

where m is constant.

We can not take the order $(-\alpha+1) \in Z^-$ because $(-\alpha+1)$ is fractional, so we will take $(-\alpha+1) \notin Z^- \cup \{0\}$,

i,e.

$$(0)_{-\alpha} = \left((0)_{-1}\right)_{-\alpha+1} = (m)_{-\alpha+1} = 0, \quad \text{if} \quad (-\alpha+1) \notin Z^- \cup \{0\}.$$

By using lemma (2) the equation (21) becomes

$$\varphi = k \cdot \left((P(Z))^{-1}\right)_{\alpha-3}, \qquad (22)$$

where k is constant.

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تطبيق حساب التفاضل والتكامل الكسرى لنيشوموتو لحل معادلة تفاضلية عادية من الرتبة الرابعة

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الملخص

العديد من الأوراق العلمية قد نشرت في نفس مجال هذا البحث بواسطة K. Nishimoto, S. Owa, Shih-Tong Tu and H. M. Srivastava.

وفي هذه الورقة العلمية تم الحصول على حل لمعادلة تفاضلية خطية عادية من الرتبة الرابعة بواسطة حساب التفاضل والتكامل الكسرى لنيشوموتو.

الكلمات المفتاحية: تفاضل وتكامل نيشوموتو, معادلة تفاضلية عادية, معادلة تفاضلية متجانسة وغير متجانسة.