On certain $P2$ −Like and P^* −Generalized $\mathcal{B}$K −Recurrent Finsler Space

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Abstract

In the present paper, we study certain types of generalized $\mathcal{B}$K-recurrent Finsler space, we shall introduce a definition for a generalized $\mathcal{B}$K-recurrent space to be $P2$ −like space and P^* − space, respectively. We shall call them $P2-$ like generalized $\mathcal{B}K$ −recurrent space and P^* −generalized $\mathcal{B}K$ −recurrent space, respectively. Different theorems concerning these spaces, we also plan to obtain some identities in these spaces.

Keywords: Finsler space, $P2-$ like generalized $\mathcal{B}K$ −recurrent space, P^* −generalized $\mathcal{B}K$ −recurrent space.

Introduction

Verma [13] discussed recurrent property of Cartan's fourth curvature tensor R^i_{jkh}. Dikshit [4] discussed birecurrent of Berwald curvature tensor H^i_{jkh}. Dwivedi [5] worked out the role of P^*- reducible space in affinely connected space. Cartan [3] introduced it as one of particular cases and further Berwald [1], [2] showed that the space was characterized by $P^i_{jkh}=0$, where P^i_{jkh} is the hv - curvature tensor. Dwivedi [5] worked out the role of P^*-reducible space in Landsberg space.

Let us consider an n-dimensional Finsler space F_n equipped with the metric function F(x,y) satisfies the request condition Rund [12].

The relation between the metric function F and the corresponding metric tensor is given by

(1.1) $$g_{ij}(x,y)=\tfrac{1}{2}\dot{\partial}_i\dot{\partial}_j F^2(x,y).$$

The tensor $g_{ij}(x,y)$is symmetric and a positively homogeneous of degree zero in y^i.

The vector y_i and its associative y^i satisfy the following relation

(1.2) $$g_{ij}(x,y)y^i=y_j.$$

The two sets of quantities g_{ij} and its associative g^{ij}, which are components of a metric tensor are connected by

(1.3) $$a)\ g_{ij}g^{ik}=\delta^k_j=\begin{cases}1 & if\ i=k,\\ 0 & if\ i\neq k\end{cases} \quad \text{and} \quad b)\ \delta^i_h g_{ik}=g_{hk}.$$

By differentiating (1.1) partially with respect to y^k, we construct a new tensor C_{ijk} defined by

$$C_{ijk}=\tfrac{1}{2}\dot{\partial}_i g_{jk}.$$

This new tensor C_{ijk} is positively homogeneous of degree -1 in $y^i$and symmetric in all its indices called *(h)hv-torsion tensor* Matsumoto [10]. According to Euler's theorem on homogeneous functions, this tensor satisfies the following:

(1.4) $$C_{ijk}y^i == C_{kij}y^i = C_{jki}y^i = 0.$$

The tensor C^i_{jk} is the associate tensor of C_{ijk} defined by

$$a)\ C_{jsk}=C^i_{jk}g_{is} \quad \text{and} \quad b)\ C_{sjk}g^{ji}=C^i_{sk}.$$

The tensor C^h_{ik} is called *(v) hv-torsion tensor*, and is positively homogeneous of degree -1 in y^i and symmetric in its lower indices, i.e.

$$C^h_{ik}=C^h_{ki}.$$

This tensor satisfies the following identities

(1.5) $C^i_{jk} y^k = C^i_{kj} y^k = 0.$

Berwald's covariant derivative of the vector y^i vanish identically, i.e.

(1.6) $\mathcal{B}_k y^i = 0.$

In general, Berwald's covariant derivative of the metric tensor g_{ij} doesn't vanish and is given by

(1.7) $\mathcal{B}_k g_{ij} = -2C_{ijk|h} y^h = -2y^h \mathcal{B}_h C_{ijk}.$

Remark 1.1. The symbol $|h$ is the covariant differential operator with respect to x^h in the sense of Cartan.

The tensor K^i_{jkh} is called *Cartan's fourth curvature tensor*, and is positively homogeneous of degree zero in y^i, defined by Rund [12]

$$K^i_{jkh} := \partial_h \Gamma^{*i}_{kj} + (\dot{\partial}_s \Gamma^{*i}_{jh}) G^s_k + \Gamma^{*i}_{th} \Gamma^{*t}_{kj} - h/k \quad .$$

Also, this curvature tensor K^i_{jkh} satisfies the following relation too

(1.8) $K^i_{jkh} = R^i_{jkh} - C^i_{jr} H^r_{kh}$

The h(v)-torsion tensor H^i_{kh}, the curvature tensors K^i_{jkh} and R^i_{jkh} are connected by Rund[12]

(1.9) $K^i_{jkh} y^j = H^i_{kh} = R^i_{jkh} y^j.$

The curvature tensor R^i_{jkh} is called *Cartan's third curvature tensor*, and is positively homogeneous of degree zero in y^i, defined by Rund [12]

$$R^i_{jkh} = \partial_h \Gamma^{*i}_{jk} + \left(\dot{\partial}_l \Gamma^{*i}_{jk}\right) G^l_h + C^i_{jm}\left(\dot{\partial}_k G^m_h - G^m_{kl} G^l_h\right) + \Gamma^{*i}_{mk} \Gamma^{*m}_{jh} - k/h.$$

The curvature tensor R^i_{jkh} satisfies the following :

(1.10) a) $R^r_{jkr} = R_{jk}$ and $b)$ $R^i_{jkh} g^{jk} = R^i_h.$

The curvature scalar *R* is given by Rund [12]

(1.11) $R_{jk} g^{jk} = R.$

The curvature vector tensor R_j is given by Rund [12]

(1.12) $R_j = K_j + C^i_{jr} H^r_i.$

The associate curvature tensor R_{ijkh} of the curvature tensor R^i_{jkh} is given by Rund [12]

(1.13) $R_{ijhk} = g_{rj} R^r_{ihk}$.

Also the curvature tensor R^i_{jkh} satisfies the following identity Rund [12]

$$R^i_{jkh|s} + R^i_{jsk|h} + R^i_{jhs|k} + y^m (R^r_{mhs} P^i_{jkr} + R^r_{mkh} P^i_{jsr} + R^r_{msk} P^i_{jhr}) = 0,$$

where P^i_{jkh} is known as *hv- curvature tensor* (*Cartan's second curvature tensor*) and is defined by Rund [12]

(1.14) $P^i_{jkh} = \dot{\partial}_h \Gamma^{*i}_{jk} + C^i_{jm} P^m_{kh} - C^i_{jh|k}$

or equivalent by

$$P^i_{jkh} = \dot{\partial}_h \Gamma^{*i}_{jk} + C^i_{jr} C^r_{kh|s} y^s - C^i_{jh|k}$$

or

$$P^i_{jkh} = C^i_{kh|j} - g^{ir} C_{jkh|r} + C^r_{jk} P^i_{rh} - P^r_{jh} C^i_{rk} .$$

The curvature tensor P^i_{jkh} is positively homogeneous of degree zero in y^i and the tensor satisfies the following:

(1.15) a) $P^i_{jkh} y^j = \Gamma^{*i}_{jkh} y^j = P^i_{kh} = C^i_{kh|r} y^r$ and b) $P^i_{jkh} y^k = P^i_{jkh} y^h = 0,$

where P^i_{kh} is called as *v(hv)- torsion tensor* and the associative tensor P_{rkh} is given by Rund [12]

(1.16) $P^i_{kh}\, g_{ir} = P_{rkh}$.

The curvature vector P_k is given by

(1.17) $P^i_{ki} = P_k$.

A Finsler space F_n for which the curvature tensor R^i_{jkh} satisfies the following Hussien [6]:

(1.18) $\mathcal{B}_m R^i_{jkh} = \lambda_m R^i_{jkh}$, $R^i_{jkh} \neq 0$

is called R^h −*recurrent space*, where λ_m is non-zero covariant vector field.

Transvecting the condition (1.18) by y^j , using (1.9) and (1.6), we get

(1.19) $\mathcal{B}_m H^i_{kh} = \lambda_m H^i_{kh}$.

Definition 1.1. A Finsler space F_n for which Cartan's fourth curvature tensor K^i_{jkh} satisfies the condition Qasem and Baleedi [11]

(1.20) $\mathcal{B}_m K^i_{jkh} = \lambda_m K^i_{jkh} + \mu_m(\delta^i_h g_{jk} - \delta^i_k g_{jh})$, $K^i_{jkh} \neq 0$,

will be called *generalized $\mathcal{B}K$-recurrent space*, where λ_m and μ_m are non-zero covariant vectors field and tensor will be *generalized recurrent tensor*. We shall denote such space and tensor briefly by $G\mathcal{B}K - RF_n$ and $G\mathcal{B}K - R$, *respectively* Qasem and Baleedi [11].

Transvecting the condition (1.20) by y^j, using (1.9), (1.6) and (1.2), we get

(1.21) $\mathcal{B}_m H^i_{kh} = \lambda_m H^i_{kh} + \mu_m(\delta^i_h y_k - \delta^i_k y_h)$.

Taking the covariant derivative for (1.8) with respect to x^m in the sense of Berwald, using the condition (1.20) and (1.21), we get

(1.22) $$\mathcal{B}_m R^i_{jkh} = \lambda_m K^i_{jkh} + \mu_m(\delta^i_h g_{jk} - \delta^i_k g_{jh}) + (\mathcal{B}_m C^i_{jr}) H^r_{kh} + C^i_{jr}[\lambda_m H^r_{kh} + \mu_m(\delta^r_h y_k - \delta^r_k y_h)].$$

Using (1.8) in (1.22), we get

(1.23) $$\mathcal{B}_m R^i_{jkh} = \lambda_m R^i_{jkh} + \mu_m(\delta^i_h g_{jk} - \delta^i_k g_{jh} + C^i_{jh} y_k - C^i_{jk} y_h) + (\mathcal{B}_m C^i_{jr}) H^r_{kh}.$$

2. A P2 − Like − G$\mathcal{B}$K − RF_n

Definition 2.1. A P2 −like space is characterized by Matsomoto[10]

(2.1) $P^i_{jkh} = \varphi_j C^i_{kh} - \varphi^i C_{jkh}$,

where φ_j and φ^i are non-zero covariant and contravariant vector fields, respectively.

Definition 2.2. The $G\mathcal{B}K - RF_n$ which is P2 − like space [satisfies the condition (2.1)] will be called $P2 - Like - G\mathcal{B}K$ −*recurrent space* and is denoted briefly by $P2 - Like - G\mathcal{B}K - RF_n$.

Transvecting (1.8) by φ_r and using (2.1), we get

(2.2) $\varphi_r R^i_{jkh} = \varphi_r K^i_{jkh} + (P^i_{rjm} + \varphi^i C_{rjm}) H^m_{kh}$.

Transvecting (2.2) by y^r, using (1.15a) and (1.4), we get

(2.3) $\varphi R^i_{jkh} = \varphi K^i_{jkh} + P^i_{jm} H^m_{kh}$,

where $\varphi = \varphi_r y^r$.

Using (1.8) in (2.3), we get

(2.4) $P^i_{jm} = \varphi C^i_{jm}$,

since $H^m_{kh} \neq 0$ .Thus, we conclude that

Theorem 2.1. *In $P2 - Like - G\mathcal{B}K - RF_n$, the torsion tensor P^i_{jm} is proportional to the torsion tensor C^i_{jm}.*

Taking the covariant derivative for (2.2) with respect to x^m in the sense of Berwald and using (1.20), we get

(2.5) $$\mathcal{B}_m(\varphi_r R^i_{jkh}) = \varphi_r \lambda_m K^i_{jkh} + \varphi_r \mu_m(\delta^i_h g_{jk} - \delta^i_k g_{jh}) + K^i_{jkh}\mathcal{B}_m \varphi_r + \mathcal{B}_m[(P^i_{rjm} + \varphi^i C_{rjm}) H^m_{kh}].$$

Using (2.2) in (2.5), we get

(2.6) $$\mathcal{B}_m(\varphi_r R^i_{jkh}) = \lambda_m(\varphi_r R^i_{jkh}) + \varphi_r \mu_m(\delta^i_h g_{jk} - \delta^i_k g_{jh}) + K^i_{jkh}\mathcal{B}_m \varphi_r + \mathcal{B}_m[(P^i_{rjm} + \varphi^i C_{rjm}) H^m_{kh}] - \lambda_m(P^i_{rjm} + \varphi^i C_{rjm}) H^m_{kh}.$$

which can be written as

(2.7) $\varphi_r \mathcal{B}_m R^i_{jkh} + R^i_{jkh}\mathcal{B}_m \varphi_r = \lambda_m(\varphi_r R^i_{jkh}) + \varphi_r \mu_m(\delta^i_h g_{jk} - \delta^i_k g_{jh})$

$$+K^i_{jkh}\mathcal{B}_m\varphi_r+\mathcal{B}_m\left[\left(P^i_{rjm}+\varphi^i C_{rjm}\right)H^m_{kh}\right]-\lambda_m\left(P^i_{rjm}+\varphi^i C_{rjm}\right)H^m_{kh}.$$

This shows that

(2.8) $$\mathcal{B}_m R^i_{jkh}=\lambda_m R^i_{jkh}+\mu_m\left(\delta^i_h g_{jk}-\delta^i_k g_{jh}\right),$$

if and only if

(2.9) $$\mathcal{B}_m\left[\left(P^i_{jkh}+\varphi^i C_{rjm}\right)H^m_{kh}\right]=\lambda_m\left(P^i_{jkh}+\varphi^i C_{rjm}\right)H^m_{kh}+R^i_{jkh}\mathcal{B}_m\varphi_r$$
$$-K^i_{jkh}\mathcal{B}_m\varphi_r,$$

since $\varphi_r\neq 0$. Assuming $\mathcal{B}_m\varphi_r=0$, i. e. covariant constant, then

$$\mathcal{B}_m\left[\left(P^i_{jkh}+\varphi^i C_{rjm}\right)H^m_{kh}\right]=\lambda_m\left[\left(P^i_{jkh}+\varphi^i C_{rjm}\right)H^m_{kh}\right].$$

Thus, we conclude that

Theorem 2.2. *In $P_2-like-G\mathcal{B}K-RF_n$, the curvature tensor R^i_{jkh} behaves as generalized recurrent if and only if the tensor $\left(P^i_{jkh}+\varphi^i C_{rjm}\right)H^m_{kh}$ behaves as recurrent, provided that $\mathcal{B}_m\varphi_r=0$.*

Transvecting (2.5) by y^j , using (1.6), (1.9), (1.4), (1.2) and (1.15b), we get

(2.10) $$\mathcal{B}_m H^i_{kh}=\lambda_m H^i_{kh}+\mu_m\left(\delta^i_h y_k-\delta^i_k y_h\right),$$

if and only if

(2.11) $$\mathcal{B}_m\varphi_r=0\ ,$$

since $H^i_{kh}\neq 0$. Thus, we conclude that

Theorem 2.3. *In $P2-Like-G\mathcal{B}K-RF_n$, Berwald's covariant derivative of first order for the hv-torsion tensor H^i_{kh} is given by (2.10) if and only if $\mathcal{B}_m\varphi_r=0$ holds good.*

Transvecting (1.12) by φ_m, we get

(2.12) $$\varphi_m R_j=\varphi_m K_j+\varphi_m C^i_{jh}H^h_i.$$

Using (2.1) in (2.12), we get

(2.13) $$\varphi_m R_j=\varphi_m K_j+\left(P^i_{mjh}+\varphi^i C_{mjh}\right)H^h_i.$$

Transvecting (2.13) by y^m, using (1.15a) and (1.4), we get

(2.14) $$\varphi_m y^m R_j=\varphi_m y^m K_j+P^i_{jh}H^h_i$$

which can be written as

(2.15) $$P^i_{jh}H^h_i=\varphi\left(R_j-K_j\right),$$

where $\varphi=\varphi_m y^m$.

Using (1.12) in (2.15), we get

(2.16) $$P^i_{jh}H^h_i=\varphi\ C^i_{jh}H^h_i$$

or

(2.17) $$P^i_{jh}=\varphi C^i_{jh}\ ,\quad \text{since } H^h_i\neq 0.$$

Transvecting (2.17) by y^h or y^j and using (1.5),we get

(2.18) a) $P^i_{jh}y^h=0$ and b) $P^i_{jh}y^j=0$.

Thus, we conclude that

Theorem 2.4. *In $P_2-Like-G\mathcal{B}K-RF_n$, we have the identities (2.15), (2.17), (2.18a) and (2.18b).*

3. An $P^*-G\mathcal{B}K-RF_n$

Definition 3.1. A P^* −Finsler space is characterized by the condition Izumi ([7], [8], [9])

(3.1) $$P^i_{kh}=C^i_{kh|j}y^j=\varphi C^i_{kh}\quad ,\ \varphi\neq 0.$$

Definition 3.2. The $G\mathcal{B}K-RF_n$ which is A P^* −Finsler space [satisfies the condition (3.1)] will be called $P^*-G\mathcal{B}K$ −*recurrent space* and is denoted briefly by $P^*-G\mathcal{B}K-RF_n$.

Transvecting (1.23) by φ and using (3.1),we get

(3.2) $$\varphi\mathcal{B}_m R^i_{jkh}=\lambda_m\varphi R^i_{jkh}+\varphi\mu_m\left(\delta^i_h g_{jk}-\delta^i_k g_{jh}\right)+\varphi\mu_m\left(P^i_{jh}y_k-P^i_{jk}y_h\right)$$
$$+\left(\mathcal{B}_m P^i_{jr}\right)H^r_{kh}.$$

This shows that

$$\mathcal{B}_m R^i_{jkh} = \lambda_m R^i_{jkh} + \mu_m(\delta^i_h g_{jk} - \delta^i_k g_{jh})$$

if and only if

(3.3) $$\varphi\mu_m(P^i_{jh} y_k - P^i_{jk} y_h) + (\mathcal{B}_m P^i_{jr}) H^r_{kh} = 0.$$

Thus, we conclude that

Theorem 3.1. *In $P^* - GBK - RF_n$, Cartan's third curvature tensor R^i_{jkh} behaves as generalized recurrent if and only if the condition (3.3)holds good.*

Transvecting (3.2) by g_{it} , using (1.3b), (1.13), (1.6) and (1.16), we get

$$\varphi\mathcal{B}_m R_{jtkh} = \varphi\lambda_m R_{jtkh} + \varphi\mu_m(g_{jk} g_{ht} - g_{jh} g_{kt} + P_{tjh} y_k - P_{tjk} y_h) \\ +(\mathcal{B}_m P_{tjr}) H^r_{kh} - \mathcal{B}_m g_{it} (P^i_{jr} H^r_{kh} - \varphi R^i_{jkh}).$$

This shows that

$$\mathcal{B}_m R_{jtkh} = \lambda_m R_{jtkh}$$

if and only if

(3.4) $$\varphi\mu_m(g_{jk} g_{ht} - g_{jh} g_{kt} + P_{tjh} y_k - P_{tjk} y_h) + (\mathcal{B}_m P_{tjr}) H^r_{kh} \\ -\mathcal{B}_m g_{it} (P^i_{jr} H^r_{kh} - \varphi R^i_{jkh}) = 0.$$

Contracting the indices i and h in the condition (3.2), using (1.3b), (1.10a) and (1.17), we get

(3.5) $$\varphi\mathcal{B}_m R_{jk} = \varphi\lambda_m R_{jk} + \varphi\mu_m[(n-1) g_{jk} + P_j y_k - P^s_{jk} y_s] + (\mathcal{B}_m P^s_{jr}) H^r_{ks}.$$

This shows that

$$\mathcal{B}_m R_{jk} = \lambda_m R_{jk}$$

if and only if

(3.6) $$\varphi\mu_m[(n-1) g_{jk} + P_j y_k - P^s_{jk} y_s] + (\mathcal{B}_m P^s_{jr}) H^r_{ks} = 0.$$

Transvecting (3.2) by g^{jk}, using (1.3a) ,(1.10b) and in view of (1.3) ,we get

$$\varphi\mathcal{B}_m R^i_h = \lambda_m \varphi R^i_h + \varphi\mu_m[(n-1)\delta^i_h + g^{jk}(P^i_{jh} y_k - P^i_{jk} y_h)] \\ +g^{jk}(\mathcal{B}_m P^i_{jr}) H^r_{kh} + \varphi(\mathcal{B}_m g^{jk}) R^i_{jkh}.$$

This shows that

$$\mathcal{B}_m R^i_h = \lambda_m R^i_h$$

if and only if

(3.7) $$\varphi\mu_m[(n-1)\delta^i_h + g^{jk}(P^i_{jh} y_k - P^i_{jk} y_h)] + g^{jk}(\mathcal{B}_m P^i_{jr}) H^r_{kh} \\ +\varphi(\mathcal{B}_m g^{jk}) R^i_{jkh} = 0.$$

Transvecting (3.5) by g^{jk}, using (1.3a) and (1.11),we get

$$\varphi\mathcal{B}_m R = \lambda_m \varphi R + \varphi\mu_m[n(n-1) + g^{jk}(P_j y_k - P^s_{jk} y_s)] + \varphi(\mathcal{B}_m g^{jk}) R_{jk} + \\ g^{jk}(\mathcal{B}_m P^i_{jr}) H^r_{kh}.$$

This shows that

$$\mathcal{B}_m R = \lambda_m R$$

if and only if

(3.8) $$\varphi\mu_m[n(n-1) + g^{jk}(P_j y_k - P^s_{jk} y_s)] + \varphi(\mathcal{B}_m g^{jk}) R_{jk} + g^{jk}(\mathcal{B}_m P^i_{jr}) H^r_{kh} \\ = 0.$$

Thus, we conclude that

Theorem 3.2. *In $P^* - GBK - RF_n$, the associative curvature tensor R_{jpkh}, Ricci tensor R_{jk}, the deviation tensor R^i_h and the curvature scalar R all behave as recurrent if and only if (3.4), (3.6), (3.7) and (3.8), respectively hold good.*

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حول فضاء فنسلر $P2$ −like و P^* − المعمم $\mathcal{B}K$− أحادي المعاودة

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الملخص

حيث تم تقديم تعريف أحادي المعاودة $\mathcal{B}K$− في هذه الورقة تم دراسة بعض الأنواع لفضاء فنسلر المعمم__ على التوالي، P^* − space وَ $P2$ − like spaceأحادي المعاودة لكي يكون فضاء $\mathcal{B}K$− فضاء فنسلر المعمم وتم تسميتهما بــ
P^* − generalized $\mathcal{B}K$ −recurrent space و $P2$ − like generalized $\mathcal{B}K$ −recurrent space
على الترتيب.
وتم الحصول على مبرهنات مختلفة، والعديد من المطابقات التي تتحقق في هذه الفضاءات.
P^* − spaceأحادي المعاودة، فضاء $\mathcal{B}K$− المعمم− $P2$ like space

الكلمات المفتاحية: فضاء فنسلر، فضاء أحادي المعاودة . $\mathcal{B}K$− المعمم