Study of the relation in the recurrent Finsler space of different orders

Fahmi Yaseen AbdoQasem[1], Gamal Abobakar Abdallah Bawazeir [1]and [2]Ali Ali Ali Muhib

[1]Dept. of Maths., Faculty of Edu.-Aden, Univ. of Aden, Khormaksar, Aden, Yemen
fahmi.yaseen@yahoo.com
[2]Dept. of Maths. , Faculty of Edu.-Thamar, Univ. of Thamar, Thamar, Yemen
muhib2005@yahoo.com
DOI: https://doi.org/10.47372/uajnas.2017.n2.a12

Abstract

In the present paper, we define a R^h-recurrent space, R^h-birecurrent space, R^h-generalized birecurrent space of the first kind, R^h-generalized birecurrent space of the second kind, R^h-special generalized birecurrent space of the first kind and R^h-special generalized birecurrent space of the second kind. The aim of this paper is to study the relation between the above spaces.

Keywords: R^h-recurrent space, R^h-birecurrent space, R^h-generalized birecurrent of the first kind, R^h-generalized birecurrent of the second kind, R^h-special generalized birecurrent of the first kind and R^h-special generalized birecurrent of the second kind.

1. Introduction

The concept of recurrent curvature of an n-dimensional Riemannian space was extended to a Finsler space by Moór([9],[10]) for the first time.Finsler space with different types of recurrent curvature tensors have been discussed bySen [19],Mishra and Pande [4],Misra([5],[6],[7]), Misra and Meher[8], Pande and Singh [11], Pandey ([12],[13],[14]), Pandey and Misra[8], Dubey and Srivastava[2], Pandey and Dwivedi[16],Verma [20], Dikshit [1] and others. Several contributions have been made by above the others authors to spaces recurrent curvature,Verma[20]and Mishra and Lodhi[3] discussed C^h -birecurrent space.We have little information about the space in which curvature tensor R^i_{jkh} is h-recurrent.

Verma[20] introduced a Finsler space in which curvature tensorR^i_{jkh} is recurrent, Dikshit [1] discussed a Finsler space in which curvature tensor R^i_{jkh} is birecurrent,Qasem[18] discussed a Finsler spaces in which Cartan's third curvature tensor R^i_{jkh}generalized (special generalized) birecurrent of the first and second kind, Qasem and Muhib[18] discussed a Finsler space in which Cartan's third curvature tensor$R^i_{jkh}$is trirecurrent .

In the present paper, we introduced a R^h -generalized trirecurrent and R^h -specialgeneralizedtrirecurrentFinslerspaces.We also discussed different Finsler spaces with recurrent of different orders for Cartan's curvature tensor R^i_{jkh}as well as studying the relation between them.

2. A R^h-Generalized BirecurrentFinsler Space

Verma [20] discussed a Finsler space in which Cartan's third curvature tensorR^i_{jkh}satisfies the recurrence condition, with respect to Cartan's connection parameterΓ^{*i}_{rk}, and called it R^h-*recurrent space*. Thus, a R^h-recurrent space is characterized by the condition

(2.1)$R^i_{jkh|\ell} = \lambda_\ell \, R^i_{jkh}$,$R^i_{jkh} \neq 0$,

where$|\ell$ is h-covariant differential operator of the first order,with respect to$x^l$and λ_ℓ, is anon-zero covariant vector field is called *recurrence vector field*.

Dikshit [1] discussed a Finsler space in which Cartan's third curvature tensorR^i_{jkh}, satisfies the birecurrence condition, with respect to Cartan's connection parameterΓ^{*i}_{rk}, and called itR^h-*birecurrent space*. Thus, a R^h-birecurrent space is characterized by the condition

(2.2)$R^i_{jkh|\ell|m} = a_{\ell m} R^i_{jkh}$,$R^i_{jkh} \neq 0$,

where $|\ell|m$ is h-covariant differential operator of the second order,with respect to x^landx^msuccessively, and$a_{\ell m}$is non-zero covariant tensor field of the second order is called *recurrence tensor field* .

Qasem [20] discussed a more general Finsler space for which Cartan's third curvature tensor R^i_{jkh} satisfies the generalized and special generalized birecurrence condition of the first and the second kind, with respect to Cartan's connection parameter Γ^{*i}_{rk}, and called them R^h-*generalized birecurrent space of the first kind,* R^h*-generalized birecurrent space of the second kind,* R^h*-special generalized birecurrent space of the first kind and* R^h*-special generalized birecurrent space of the second kind*. They are characterized by the condition

(2.3) a) $R^i_{jkh|\ell|m} = \lambda_\ell R^i_{jkh} + b_{\ell m} R^i_{jkh}$,

b) $R^i_{jkh|m|\ell} = \lambda_m R^i_{jkh|\ell} + b_{\ell m} R^i_{jkh}$

and

(2.4) a)$R^i_{jkh|\ell|m} = \lambda_\ell R^i_{jkh|m}$,

b)$R^i_{jkh|m|\ell} = \lambda_m R^i_{jkh|\ell}$

respectively, where $R^i_{jkh} \neq 0$, $|\ell|m$ is h-covariant differential operator of the second order,with respect to x^l and x^m successively,also λ_r and $b_{\ell m}$ are anon-zero covariant tensor field and covariant tensor field of second order,respectively.

Let us consider a R^h-recurrent Finsler space characterized by the condition (2.1).

Taking the h-covariant derivative for the condition (2.1) with respect to x^m, we get

$R^i_{jkh|\ell|m} = \lambda_{\ell|m} R^i_{jkh} + \lambda_\ell R^i_{jkh|m}$,$R^i_{jkh} \neq 0$

which can be written as

(A) $R^i_{jkh|\ell|m} = \lambda_\ell R^i_{jkh|m} + b_{\ell m} R^i_{jkh}$,$R^i_{jkh} \neq 0$,

where $\lambda_\ell$is non-zero covariant vector fieldand $b_{\ell m} = \lambda_{\ell|m}$ is non-zerocovariant tensor field of the second order.

Thus, we may conclude

Theorem 2.1. Every R^h*-recurrent spaceis* R^h*-generalized birecurrentspace.*

Now, in view of the condition (2.1), the condition (A) may be written as

$R^i_{jkh|\ell|m} = \lambda_\ell \lambda_m R^i_{jkh} + b_{\ell m} R^i_{jkh}$,$R^i_{jkh} \neq 0$

which can be written as

$R^i_{jkh|\ell|m} = a_{\ell m} R^i_{jkh}$,$R^i_{jkh} \neq 0$,

where$a_{\ell m} = \lambda_\ell \lambda_m + b_{\ell m}$ is the non-zero covariant vector field of second order.

Thus, we may conclude

Theorem 2.2.*In* R^h *-recurrent spacethe* R^h *-generalized birecurrent space is* R^h *-birecurrentspace.*

In view of the condition (2.4), a R^h-special generalized birecurrent space is characterized by the condition

(B) $R^i_{jkh|\ell|m} = \lambda_m R^i_{jkh|\ell}$,$R^i_{jkh} \neq 0$,

where$\lambda_\ell$is non-zero covariant vector field.

Remark 2.1. In view of the conditions (A) and (B), we can say that Qasem [17] consider the condition (B) as a particular case of the condition (A).

3. R^h-Generalized TrirecurrentFinsler Space

Qasem and Muhib[18] discussed a Finsler space in which Cartan's third curvature tensor R^i_{jkh} satisfies the trirecurrence condition, with respect to Cartan's condition parameterΓ^{*i}_{rk}, and called it R^h-*trirecurrent space*.

Thus, a R^h-trirecurrent spaceis characterized by the condition

(3.1) $R^i_{jkh|\ell|m|n} = a_{\ell mn} R^i_{jkh}$,$R^i_{jkh} \neq 0$,

where$a_{\ell mn}$is non-zero covariant tensor field of the third order called *trirecurrence tensor field*.

Now, taking the h-covariant derivative for the condition (A) with respect to x^n, we get

$R^i_{jkh|\ell|m|n} = \lambda_{\ell|n} R^i_{jkh|m} + \lambda_\ell R^i_{jkh|m|n} + b_{\ell m|n} R^i_{jkh} + b_{\ell m} R^i_{jkh}$,$R^i_{jkh} \neq 0$.

In view of the condition (2.1), the above equation may be written as

$R^i_{jkh|\ell|m|n}$ =$(\lambda_{\ell|n}\lambda_m$+$b_{\ell m|n} + b_{\ell m}\lambda_n)R^i_{jkh} + \lambda_\ell R^i_{jkh|m|n}$, $R^i_{jkh} \neq 0$

which can be written as

(3.2)$R^i_{jkh|\ell|m|n} = a_{\ell mn} R^i_{jkh} + \lambda_\ell R^i_{jkh|m|n}$,$R^i_{jkh} \neq 0$,

where λ_ℓ is non-zero covariant vector field and $a_{\ell mn} = \lambda_{\ell|n}\lambda_m + b_{\ell m|n} + b_{\ell m}\lambda_n$ is non-zerocovariant tensor field of third order .

Definition 3.1. AFinsler spaceF_n for which Cartan's third curvature tensor R^i_{jkh} satisfies the condition (3.2), where $\lambda_\ell$and $b_{\ell mn}$are non-zero covariant vector field and covariant tensor field of third order respectively, will be called R^h-*generalized trirecurrent space* and the tensor by *h-generalized trirecurrent tensor*. We shall denote such space and tensor briefly by R^h-*GTR*-$F_n$and *h-GTR*, respectively.

In view of the conditions (A), (2.1) and(3.2), we may conclude

Theorem3.1.*In* R^h-*recurrent space,the*R^h-*generalized birecurrent space is*R^h-*generalized birecurrent space*.

Now, taking the h-covariant derivative for the condition (B) with respect to x^n, we get

(C) $R^i_{jkh|\ell|m|n} = \lambda_{m|n} R^i_{jkh|\ell} + \lambda_m R^i_{jkh|\ell|n}$,$R^i_{jkh} \neq 0$.

In view of the condition (2.1), the condition (C) may be written as

$R^i_{jkh|\ell|m|n} = \lambda_{m|n}\lambda_\ell R^i_{jkh} + \lambda_m R^i_{jkh|\ell|n}$,$R^i_{jkh} \neq 0$

which can be written as

$R^i_{jkh|\ell|m|n} = a_{\ell mn}R^i_{jkh} + \lambda_m R^i_{jkh|\ell|n}$,$R^i_{jkh} \neq 0$,

where $\lambda_m$is non-zero covariant vector fieldand $a_{\ell mn} = \lambda_{m|\ell}\lambda_\ell$ is non-zero covariant tensor field of third order .

Thus, we may conclude

Theorem 3.2. *In* R^h *-recurrent space,the* R^h *-special generalized birecurrent space is* R^h *-generalized trirecurrent space*.

Definition 3.2. A Finsler space for which Cartan's third curvature tensor R^i_{jkh} satisfies the condition

(3.3)$R^i_{jkh|\ell|m|n} = \lambda_m R^i_{jkh|\ell|n}$,$R^i_{jkh} \neq 0$,

where$\lambda_\ell$is non-zero covariant vector field. The space and the tensor satisfy the condition(3.3) and will be called R^h *-special generalized trirecurrent space* and *h-special generalized trirecurrent tensor,* respectively. We shall denote such space and tensor briefly by R^h-*SGTR*-$F_n$and*h-SGTR*, respectively.

In view of the conditions (B) and (C), we may conclude

Theorem3.3.*Every R^h -special generalized birecurrent spaceis R^h -special generalized trirecurrent space if the covariant derivative of the covariant vector in the sense of Cartanvanishes .*

Remark 3.1. The condition (3.3) looks as a particular case of the condition (3.2).

References

1. Dikshit, S. (1992): *Certain types of recurrences in Finsler spaces*, D. Phil. Thesis, University of Allahabad, (Allahabad), (India), 26 - 109.
2. Dubey, R.S.D. and Srivastava, A.K.(1981): On recurrent Finsler spaces, Bull. Soc. Math. Belgique, 33, 283-288.
3. Mishra, C. K. and Lodhi, G.(2008):On C^h -recurrent and C^v -recurrent Finslerspaces of second order, Int. J. Contemp. Math. Sciences, Vol. 3, No. 17, 801-810.
4. Mishra,R.S. and Pande,H.D.(1986):Recurrent Finsler spaces, J. Indian Math. Soc. N.S. 32,17-22.
5. Misra, R.B.(1977):A turning point in the theory of recurrent Finsler manifold,J.South Gujarat Univ.,6, 72-96.
6. Misra,R.B.(1979):A turning point in the theory of recurrent Finsler manifold II. Certain type of projective motions, Bull, Un., Mat. Ital.(5), 8-16.
7. Misra,R.B.(1973):On recurrent Finsler space, Rev, Roumaine .Math .Purse Math. Kyoto Univ.,18, 701-712.
8. Misra, R.B.and Maher,F.M.(1973):A recurrent Finsler space of second order, Rev. Roumaine, Math.Purse, Appl.18, 563-569.
9. Moór, A. (1972): Unterräume von rekurrenter krümmung in Finslerrmen, Tensor N.S., 24, 261-265.
10. Moór, A. (1963): Untersushngen $\ddot{u}$ber Finslerräume von rekurrenter krümmung, Tensor N.S., 13, 1-18.
11. Pande, H.D. and Singh, B. (1977): On existence of the affinely connected Finsler space with recurrent tensor field, reprinted from Indian jour. of Pure and Appl. Math., Vol.8, No.3, (March 295-301.
12. Pandey, P.N.(1981): A note on recurrence vector, proc. Nat.Acad.Sci.(India), 51A, l, 6-8.
13. Pandey, P.N. (1979):A recurrent Finsler manifold admitting special recurrent transformations,Prog.Math.13(1-2), 85-98.
14. Pandey, P.N. (1983): On Lie recurrent Finsler manifold, Indian J. Mathematics, 24, 135-143.
15. Pandey, P.N.and Dwivedi,V.J. (1987):On T-recurrent Finsler spaces, Prog. Math., 21(2), 101-112.
16. Pandey,P.N and Misra,R.B. (1981):Projective recurrent Finsler manifolds I, Publications Mathematics Debrecen, 28(3-4), 191-198.
17. Qasem, F.Y.A. (2000): *On transformations in Finsler spaces*, D. Phil, Thesis, University of Allahabad, (Allahabad), (India), 26 – 101.
18. Qasem, F.Y.A. and Muhib, A.A.A. (2008):On R^h-TrirecurrentFinsler spaces, The Scientific Journal of The Faculty of Education-Thamar, University of Thamar, Vol.(1), No.(5), 59-75.
19. Sen, R.N. (1968):Finsler spaces of recurrent curvature, Tensor N.S., 19, 291–299.
20. Verma, R. (1991):Some transformation in Finsler spaces, D. Phil. Thesis University of Allahabad, (Allahabad), (India), 21-59.

دراسة العلاقة في فضاء فنسلر أُحادي المُعاودة ذات مراتب مُختلفة

[1]فهمي ياسين عبده قاسم ، [1]جمال أبوبكر عبدالله باوزير و[2]علي علي علي مُحب
[1]قسم الرياضيات، كلية التربية - عدن، جامعة عدن ، خورمكسر ، عدن ، اليمن.
Fahmi.yaseen@yahoo.com
[2]قسم الرياضيات، كلية التربية - ذمار، جامعة ذمار ، ذمار ، اليمن.
Muhib2005@yahoo.com
DOI: https://doi.org/10.47372/uajnas.2017.n2.a12

الملخص

في الورقة الحالية، نُعرف فضاءR^h– أحادي المعاودة ، فضاء R^h– ثُنائي المُعاودة، فضاء R^h– تعميم ثُنائي المُعاودة من النوع الأول، فضاء R^h- تعميم ثُنائي المُعاودة من النوع الثاني، فضاء R^h- خاص تعميم ثُنائي المُعاودة من النوع الأول وفضاء R^h– خاص تعميم ثُنائي المعاودة من النوع الثاني , الفكرة لهذه الورقة هي دراسة العلاقة بين الفضاءات أعلاه.

الكلمات المفتاحية: فضاء R^h-أحادي المعاودة، فضاء R^h- ثُنائي المُعاودة، فضاء R^h-تعميم ثُنائي المُعاودة من النوع الأول، فضاء R^h- تعميم ثُنائي المُعاودة من النوع الثاني ، فضاء R^h- خاص تعميم ثُنائي المعاودة من النوع الأول وفضاء R^h- خاص تعميم ثُنائي المُعاودة من النوع الثاني.