A new theoretical formulation of electrical parameters related to photovoltaic cell maximum power output

Ahmed Saeed AL-Noban

Electronics and Communication Engineering Department, Faculty of Engineering, University of Aden,

Yemen

DOI: <https://doi.org/10.47372/uajnas.2016.n1.a11>

Abstract

 This work deals with a new theoretical analysis and formulation of electrical parameters lasts to a derivation of a satisfied simple maximum power output equation of photovoltaic (pv) cells. Essentially, this work concentrates on useful parameters in characterizing the maximum power output cannot be expressed explicitly in terms of other parameter. These are such as: the maximum voltage, maximum current and, hence, the maximum power output. An air mass radiation (AM1.5) is used throughout this work. First, referring to AM1.5 sunlight power versus wavelength, all the electrical parameters of wide range pv cells having band gaps ranged (0.4887 ev to 3.815 ev) are calculated by conventional way in order to compare their values with approach values. Second; instead of using the conventional way of modeling each pv cell independently, this approach introduces a method through which the modeling of several pv cells dependently is possible for the determination of the unknown electrical parameters. As a result of this analytical formulation, a satisfied derived equations of electrical parameters are obtained and satisfied up to around 100 % with no interpolation methods or other technique used.

Keywords: PV cells, PV cell electrical parameters.

Introduction

 Photovoltaic cells differ from each other according to their different band gap energy and , accordingly, their electrical output parameters differ in their values from pv cell to another according to the semiconducting material from which these photovoltaic cells are fabricated provided that the surface area of these photovoltaic cells, sunlight irradiance, and temperature under which all these different pv cell types are operated, both are identical [17, 4]. The pv cell performance is determined by its related parameters such as; the reverse saturation current (I_0) , short circuit current (I_{SC}), open circuit voltage (V_{OC}), fill factor (FF), maximum power (P_m), and conversion efficiency (η). As a material dependent parameter depends on the band gap (E_g) of the material, reverse saturation current is the critical parameter affecting the power output and, hence, the efficiency of pv cells [14, 3]. Electrical parameters are irradiance, temperature dependent parameters and, as a result, their values varies accordingly [1, 16]. Since pv cells are a major element of pv power systems, this accounts for the necessity to study the current-voltage and power-voltage characteristics of pv cells in order to obtain the maximum power output when there are changes, both, in the pv modules themselves and in the environment where they operate [13]. Relationships referring to electrical parameter, some of them cannot be expressed explicitly in terms of other parameters like maximum voltage, maximum current, and maximum power output, while some of them can be solved directly by direct substitutions into their related equations such as the short circuit current, open circuit voltage, reverse saturation current. Electrical parameters cannot be explicitly expressed in terms of other related parameters utilizing techniques such as interpolations, Newton and Raphson method in order to solve for.

 Some approximate expressions are putdown for determining some electrical parameters of pv cells, such relations are the open circuit voltage as a function of band gap energy [2]. In addition, although the fill factor of a solar cell is a useful parameter in characterizing the cell performance, it cannot be expressed explicitly in terms of other parameters. An approximate empirical expression of fill factor, is introduced, applicable for ideal cases where no parasitic resistance losses are

considered and it is accurate to about one digit in the fourth decimal place for these cases, it is an expression of fill factor as a function only of the open circuit voltag [5, 6].

Because of the difficulty of finding out some parameters, such as V_m , in addition to approximate relations, many modeling methods and techniques have been used such as the computer simulation modeling and algorithms, Newton & Raphson method, math-cad software package for the estimation of electrical parameters under different irradiance and temperature situations [13, 11, 12, 9, 7, 8]. A classical single diode modeling, using Mathcad software packages, are used to model the electrical characteristics of pv cells such that a pv cell current (I) is calculated as a function of the pv cell voltage (V) and solar radiation power (P_{in}) [13]. Laplace integral transform technique is also applied to estimate the efficiency of pv cell as a function of temperature [11]. A new approach for estimating the one-diode model parameters of a pv cell according to the irradiance and temperature is introduced, these parameters are given at known irradiance and temperature from the knowledge of I_{SC} , V_{OC} , and P_m , using Newton and Raphson method to calculate the model parameters [12]. A formula that describes the I-V characteristics is found, based on the information gathered, and the values of I and P are determined according to different values of V, afterwards, the natural cubic spline interpolation method is used to build a mathematical model that can approximate those values and finally a bisection is used as an optimization method in determining the value of V that can produce the maximum power [9]. A simplified algorithm predicts the average steady-state temperature of pv cells has been developed [7]. A simple silicon pv cell has been simulated, using MATLAB, for estimating the pv cell performance [8].

 Regardless each of the techniques and methods mentioned above, this work introduces a simple excellent approximate electrical parameter derived equations (direct-solvable equations) by analytically modeling a wide range of pv cells such that any unknown electrical parameter can easily be obtained from the known parameters certain photovoltaic cells group net (covering this wide range of pv cells) provides. This is the only way of overcoming utilizing interpolations and complex software packages. This is achieved by modeling and expressing (dependently) the unknown parameter as a function of several other known parameters for the unknown electrical parameter to be determined rather than conventionally (independently) expressed. As a result, this work presents many important equations such as the one that solves (in inverse way) for the short circuit from the open circuit voltage equation and hence the photon flux is determined. This is achieved knowing only the band gap of the considered pv cell; also the one that solves for the temperature required to equate two maximum voltages of two wide band gap different pv cells, etc.

Conventional calculations of electrical parameters of pv cell

 The conventional calculations are conducted referring to the pv cell electrical parameter equations given below:

1- The input sunlight power (P_{in}), in mW cm⁻², integrated over all wavelengths comprising the intensity spectrum of the light incident upon the pv cell is given by $[10, 8, 15]$:

$$
P_{in} = \int_0^\infty P_{in}(\lambda) d\lambda \tag{1}
$$

where, the relation connecting the photon flux (N_{ph}), in cm⁻²s⁻¹, with input sunlight power is given by:

$$
(q/hc)\int_0^\infty P_{in}(\lambda)\lambda \ d\lambda = q \int_0^\infty N_{ph}(\lambda) \ d\lambda \tag{2}
$$

Where "h" is the plank's constant, "q" is the electron charge, and "c" is the velocity of light.

2- The short circuit current density (J_{SC}) in mA cm⁻² and short circuit current (I_{SC}) in mA, generated when photons energy greater or equal the pv cell band gap energy (i.e, $hv \ge E_g$) are given by $\lceil 10, 4 \rceil$:

$$
J_{SC}(E) = qN_{ph}(E) \text{ , and; } I_{SC}(E) = qAN_{ph}(E) \tag{3}
$$

Where "A" is the cross-sectional area of the pv cell.

3- The reverse saturation current (I_0) is expressed as $[4, 14, 8]$, where " k " is the Boltzmann constant, "T" is the pv cell temperature:

 $I_0 = CT^3 e^{-\left(\frac{q}{k}\right)}$ $\frac{q}{k}$)(E_g/T) (4) Where, C is a material constant in mA /cm²/K³, and hence $I_0 = 6.03 \times 10^9 e^{-(q/k)(Eg/T)}$ mA $[4, 14]$:

4- The I-V characteristic equation that relates pv cell current (I) and voltage (V) with each other is given by:

$$
I = I_0 \left(e^{qV/kT} - 1 \right) - I_L \tag{5a}
$$

Such that; when in eq.(5a) $V = 0$, then I = -I_L referred to " short -circuited condition, where "I_L" is the light-generated current, thus:

 $I = I_{SC} = -I_{L}$ (5b) On the other hand, when in eq.(5a), $I = 0$, then this case is referred to " the open- circuit condition, and therefore, the open circuit voltage is expressed by:

$$
V_{OC} = \frac{KT}{q} \ln\{(I_{SC} + I_0) / I_0\}
$$
 (5c)

- 5- The maximum voltage (V_m) is expressed as given below: $e^{(q/k)(V_m/T)}\{1+(qV_m/kT)\} = \{(I_{SC}+I_0)/$ (6)
- 6- The maximum current (V_m) is related to the maximum voltage, by:

$$
I_{m} = \left\{ \frac{\left(\frac{qV_{m}}{kT}\right)}{\left\{1 + \left(\frac{qV_{m}}{kT}\right)\right\}} \right\} \{I_{SC} + I_{0}\}\n\tag{7}
$$

7- The maximum power (P_m) is the product of V_m and I_m , i.e: $P_m = (V_m)(I_m)$ (8) 8- The fill factor (FF) is expressed by:

$$
FF = P_m/(V_{0C} I_{SC}) = (V_m)(I_m)/(V_{0C} I_{SC})
$$

9- The voltage factor (VF) is expressed by: (9a)

$$
VF = V_{OC}/E_g
$$
 (9 b)

10- Finally, the conversion efficiency
$$
(\eta)
$$
 is given by:

$$
P_{m}/P_{in} = \frac{(V_{m})(I_{m})}{P_{in}} = \{ (V_{OC} I_{SC})(FF) \} / P_{in}
$$
 (10)

 Figure (1) shows, according to priority, the steps of calculating the electrical parameters of pv cells.

Approach theoretical modeling and formulation analysis

 Throughout this work, it is assumed that the pv cell band, covered by the wide number of pv cells, is divided into two groups; first, the group of N-pv unknown parameter pv cells: cell_{Ui} = [cell_{U1}, cell_{U2}, cell_{U3}, ….., cell_{UN}] their referred related electrical parameters are $(I_{0,\text{ui}}, I_{\text{SC,ui}}, V_{\text{m,ui}},$ $P_{m,ui}$, etc); second, the group of M-pv known (reference) parameters pv cells: cell_{Ri} = [cell_{R1}, cell_{R2}, cell_{R3},..., cell_{RM}] their referred related electrical parameters are $(I_{0,Ri}, I_{SC,Ri}, V_{m,Ri}, P_{m,Ri}$, etc), where " i " is an integer, from 1 to N and from 1 to M that indicates the pv cell number cell_{Ui} and cell_{Ri}, respectively.

Pv cells (cell_{Ui} and cell_{Ri}), both, forming a net so that any unknown parameter of any cell_{Ui} can be determined as a function of one or several cell_{Ri} for the same parameter whatever is the sunlight irradiance or the thermal conditions. This approach, general pv cells-net structure of both groups (cell_{Ui} and cell_{Ri}) is shown in Figure (2).

Approach calculations are conducted referring to the pv cell electrical parameter equations derived as given below:

Formulation of reverse saturation current (I_0 **) and band gap energy (** E_g **)**

Referring to eq.(4), a system of reverse saturation current representing, both: cell_{Ui} and cell_{Ri}, can then be written as:

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$$
I_{0,12} = CT^3 e^{-\left(\frac{q}{k}\right)(E_{g,12}/T_{,12})}
$$
(13)

$$
I_{0,R2} = CT^3 e^{-\left(\frac{q}{k}\right)(E_{g,R2}/T_{R2})}
$$
 (14)

$$
I_{0,\text{uN}} = CT^3 e^{-\left(\frac{q}{k}\right)(E_{\text{g,uN}}/T_{,\text{uN}})}
$$
(15)

$$
I_{0,RM} = CT^3 e^{-\left(\frac{q}{k}\right)(E_{g,RM}/T_{,RM})}
$$
\n
$$
(16)
$$

Multiplying eqs.(11 through 16) in themselves, results into:

 $($ $I_{0, u1}$ $I_{0, u2}$ $I_{0,N}$ $)($ $I_{0,R1}$ $I_{0,R2}$ $I_{0,M}$ $)$ = $CT^{3(N+M)}e^{-(q/k)[(E_{g, u1}/T_{u1})+(E_{gR1}/T_{R1})+(E_{gu2}/T_{u2})+(E_{gR2}/T_{R2})+\cdots+(E_{gRM}/T_{RM})+\cdots+(E_{guN}/T_{uN})]}$ (17) Taking natural logarithm for both sides of eq.(17), gives: $[(F_1/T_1) + (F_2/T_2) + (F_3/T_3) + (F_4/T_2) + (F_5/T_2) + ... + (F_n/T_n) + ...]$

()] () 2 ()() . () / 3 (18)

Arranging eq.(18):

$$
\sum_{i=1}^{N} \{ \left(E_{\text{gui}} / T_{\text{ui}} \right) \} + \sum_{i=1}^{M} \{ \left(E_{\text{gR}i} / T_{\text{R}i} \right) \} = \left(\frac{k}{q} \right) \ln \left\{ \frac{\left(C T^{3(N+M)} \right)}{\left[\left(\prod_{i=1}^{N} I_{\text{out}} \right) \left(\prod_{i=1}^{M} I_{\text{OR}i} \right) \right]} \right\}
$$
(19)
Multinlying both sides of eq. (18.) by T₁ and separate E₁ to one side gives:

Multiplying both sides of eq.(18) by T_{ul} and separate $E_{g,ul}$ to one side, gives:

$$
E_{g,u1} = \left(\frac{kT_{,u1}}{q}\right) \ln \left\{ \frac{\left(CT^{3(N+M)}\right)}{\left[\left(\prod_{i=1}^{N} I_{\text{out}}\right)\left(\prod_{i=1}^{M} I_{\text{OR}i}\right)\right]} \right\} - \left\{ \left[\left(E_{g,R1}\right) / \left(T_{,u1}/T_{,R1}\right)\right] + \left[\left(E_{g,u2}\right) / \left(T_{,u1}/T_{,u2}\right)\right] + \left[\left(E_{g,R2}\right) / \left(T_{,u1}/T_{,R2}\right)\right] + \dots + \left[\left(E_{g,uN}\right) / \left(T_{,u1}/T_{,RN}\right)\right] \right\} \tag{20}
$$

Assume a single pv cell, (cell_{u1}) out of the whole N-cell_{Ui} should communicates, photovoltaically, with all the M-cell_{Ri}. Then eq. (20) becomes:

$$
E_{g, u1} = \left(\frac{kT_{,u1}}{q}\right) \ln \left\{ \frac{\left(CT^{3(1+M)}\right)}{\left[\left(I_{0, u1}\right)\left(\prod_{i=1}^{M} I_{0, Ri}\right)\right]} \right\} - \left\{ \left[\left(E_{g, R1}\right) / \left(T_{,u1}/T_{,R1}\right)\right] + \left[\left(E_{g, R2}\right) / \left(T_{,u1}/T_{,R2}\right)\right] + \dots + \dots + \left[\left(E_{g, RM}\right) / \left(T_{,u1}/T_{,RM}\right)\right] \right\}
$$

$$
= \left(\frac{kT_{,u1}}{q}\right) \ln \left\{ \frac{\left(CT^{3(1+M)}\right)}{\left[\left(I_{0, u1}\right)\left(\prod_{i=1}^{M} I_{0, Ri}\right)\right]} \right\} - \sum_{i=1}^{M} \left[\left(E_{g, Ri}\right) / \left(T_{,u1}/T_{, Ri}\right)\right] \tag{21}
$$

Any pair out of (cell_{Ui} and cell_{Ri}) can satisfy the case. Let the pair be (cell_{U1} and cell_{R1}), then eq.(21) can be written as:

$$
E_{g,u1} = \left(\frac{kT_{u1}}{q}\right) \ln \left\{ \frac{\left(CT^{3(2)}\right)}{\left[\left(I_{0,u1}\right)\left(I_{0,R1}\right)\right]} \right\} - \left\{ \left[\left(E_{g,R1}\right) / \left(T_{u1}/T_{R1}\right)\right] \right\} \tag{22}
$$

Also, it can be proved that:

$$
E_{g,u} = E_{g,R} (T_{u}/T_{R}) + (kT_{u}/q) \ln \{ I_{0,R} / I_{0,u} \}
$$

FORMULATION OF THE OPEN CIRCUIT VOLTAGE (V_{oc}) (23)

The open circuit voltage and its negative (to use later) for cell_{Ui} and cell_{RI} pv cells electrical parameters are, respectively, given by:

$$
V_{OC,ui} = \frac{kT_{u,i}}{q} \ln \{ (I_{SC,ui} + I_{0,ui}) / I_{0,ui} \}
$$
 (24)

$$
-V_{\text{OC},ui} = \frac{k_{\text{T}_{u,i}}}{q} \ln\left\{ I_{0,ui} / (I_{\text{SC},ui} + I_{0,ui}) \right\}
$$
 (25)

$$
V_{\text{OC,Ri}} = \frac{kT_{\text{R,i}}}{q} \ln\left\{ \left(I_{\text{SC,Ri}} + I_{0,\text{Ri}} \right) / I_{0,\text{Ri}} \right\} \tag{26}
$$

$$
-V_{\text{OC,Ri}} = \frac{N_{\text{Ri}}}{q} \ln\left\{ I_{0,\text{Ri}} / (I_{\text{SC,Ri}} + I_{0,\text{Ri}}) \right\}
$$
 (27)
Same as previous analysis procedure, multiplying eqs. (24 and 26), the formulation of the open

The analysis procedure, multiplying eqs.(24 and 26), the formulation of the open circuit voltage results into:

 $V_{\text{OC},u1} V_{\text{OC},R1} V_{\text{OC},u2} V_{\text{OC},R2} ... V_{\text{OC},uN} ... V_{\text{OC},RN} =$ $\prod_{i=1}^N$ $\frac{k}{i}$ $\frac{N}{i=1} \frac{kT_{u,i}}{q} \ln \left\{ \left(I_{SC,ui} + I_{0,ui} \right) / I_{0,ui} \right\} \right\} \left\{ \prod_{i=1}^{M} \frac{k}{q} \right\}$ $_{i=1}^{M} \frac{R I_{R,i}}{q} \ln \left\{ \left(I_{SC,Ri} + I_{0,Ri} \right) / I_{0,Ri} \right\}$ (28) **FORMULATION OF THE MAXIMUM VOLTAGE (V^m)**

The maximum voltage (V_m) is expressed as:

 $e^{(q/k)(V_m/T)}\{1+(qV_m/kT)\} = \{(I_{SC} + I_0)/$ $\}$ (29) Re-arranging eq(29), yields :

$$
e^{(q/k)(V_m/T)} = \left\{ \frac{\left[\left(\frac{KT}{q}\right) + (V_m)\right]}{\left(\frac{KT}{q}\right)} \right\} \{ (I_{SC} + I_0) / I_0 \}
$$
(30)

A system of maximum voltage equations, including N-pv cell_{Ui} and M-pv cell_{Ui} can then be written as follows: T/T \sim \sim \sim

$$
e^{(q/k)\left(V_{m,u1}/T_{u1}\right)} = \left\{ \frac{\left\lfloor \left(\frac{K\cdot I_{u1}}{q}\right) + (V_{m,u1})\right\rfloor}{\left(\frac{KT_{u1}}{q}\right)} \right\} \left\{ \left(I_{SC,u1} + I_{0,u1}\right) / I_{0,u1} \right\} \tag{31}
$$

$$
e^{(q/k)\left(V_{m,R1}/T_{,R1}\right)} = \left\{ \frac{\left[\left(\frac{KT_{,R1}}{q}\right) + \left(V_{m,R1}\right)\right]}{\left(\frac{KT_{,R1}}{q}\right)}\right\} \left\{ \left(I_{SC,R1} + I_{0,R1}\right) / I_{0,R1} \right\} \tag{32}
$$

$$
e^{(q/k)\left(V_{m,u2}/T_{,u2}\right)} = \left\{ \frac{\left[\left(\frac{KT_{,u2}}{q}\right) + \left(V_{m,u2}\right)\right]}{\left(\frac{KT_{,u2}}{q}\right)}\right\} \left\{ \left(I_{SC,u2} + I_{0,u2}\right) / I_{0,u2} \right\} \tag{33}
$$

$$
e^{(q/k)\left(V_{m,R2}/T_{R2}\right)} = \left\{ \frac{\left[\left(\frac{KT_{,R2}}{q}\right) + \left(V_{m,R2}\right)\right]}{\left(\frac{KT_{,R2}}{q}\right)}\right\} \left\{ \left(I_{SC,R2} + I_{0,R2}\right) / I_{0,R2} \right\} \tag{34}
$$

$$
e^{(q/k)(V_{m,uN}/T_{uN})} = \left\{ \frac{\left[\left(\frac{KT_{uN}}{q} \right) + (V_{m,uN}) \right]}{\left(\frac{KT_{uN}}{q} \right)} \right\} \left\{ \left(I_{SC,uN} + I_{0,uN} \right) / I_{0,uN} \right\} \tag{35}
$$

$$
e^{(q/k)\left(V_{m,RM}/T_{,RM}\right)} = \left\{ \frac{\left[\left(\frac{KT_{,RM}}{q}\right) + \left(V_{m,RM}\right)\right]}{\left(\frac{KT_{,RM}}{q}\right)}\right\} \left\{ \left(I_{SC,RM} + I_{0,RM}\right) / I_{0,RM} \right\}
$$
(36)
Dividing eqs.(31 through 36), gives:

$$
e^{(\frac{q}{k})[\frac{V_{m, u1}}{T_{u1}} - \frac{V_{m, R1}}{T_{R1}} - \frac{V_{m, u2}}{T_{u2}} - \frac{V_{m, R2}}{T_{R2}} - \dots - \frac{V_{m, uN}}{T_{uN}} - \dots - \frac{V_{m, RM}}{T_{RM}}]}
$$
 =

$$
\left\{ \left\{ (I_{SC,11} + I_{0,11})/I_{0,11} \right\} \left\{ \frac{\left(\frac{KT_{11}}{q}\right)}{\left(V_{m,11}\right) + \left(\frac{KT_{11}}{q}\right)} \right\} \right\} \left\{ \left\{ (I_{0,R1})/I_{0,12} + I_{0,R2} \right\} \right\} \left\{ \left\{ (I_{0,R2})/I_{SC,12} + I_{0,12} \right\} \right\} \left\{ \left\{ \left(I_{0,R2} + I_{0,12} \right) \right\} \left\{ \frac{\left(\frac{\left(KT_{1R1}}{q}\right) + \left(V_{m,R1}\right)\right)}{\left(\frac{KT_{1R2}}{q}\right)}\right\} \right\} \left\{ \left\{ (I_{0,12})/I_{SC,12} + I_{0,12} \right\} \right\} \left\{ \left\{ (I_{0,R2})/I_{SC,12} + I_{0,12} \right\} \right\} \left\{ \left\{ (I_{0,R2})/I_{SC,12} + I_{0,12} \right\} \right\} \left\{ \left\{ (I_{0,R2})/I_{C} + I_{0,12} \right\} \right\} \left\{ \left\{ (I_{0,R2})/I_{C} + I_{0,12} \right\} \right\} \left\{ \left\{ (I_{0,R1})/I_{C} + I_{0,12} \right\} \right\} \left\{ \left\{ (I_{0,R1})/I_{C} + I_{0,12} \right\} \right\} \left\{ \left\{ (I_{0,R1})/I_{SC,RM} + I_{0,12} \right\} \right\} \left\{ \left\{ (I_{0,R2})/I_{SC,RM} + I_{0,12} \right\} \right\} \left\{ \left\{ (I_{0,R1})/I_{SC,RM} + I_{0,12} \right\} \right\} \left\{ \left\{ (I_{0,R2})/I_{SC,RM} + I_{0,12} \right\} \right\} \left\{ \left\{ (I_{0,R1})/I_{SC,RM} + I_{0,12} \right\} \right\} \left\{ \left\{ (I_{0,R2})/I_{SC,RM} + I_{0,12} \right\} \right\} \left\{ \left\{ (
$$

degree of extent nearer to unity so it is valid to substitute V_{OC} for V_m in eq.(37), and after taking ln for both its sides, eq.(37) become:

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$$
\frac{v_{m, u1}}{T_{u1}} - \frac{v_{m, R1}}{T_{u2}} - \frac{v_{m, u2}}{T_{u2}} - \frac{v_{m, R2}}{T_{u2}} - \dots - \frac{v_{m, uN}}{T_{uN}} - \dots - \frac{v_{m, RM}}{T_{uN}} = \left(\frac{k}{q}\right) \ln \left\{ \left(\left(I_{SC, u1} + I_{0, u1} \right) / \left(I_{SC, u1} \right) + I_{0, u1} \right) \right\}
$$
\n
$$
I_{0, u1} \left\{ \frac{\left(\frac{KT_{u1}}{q}\right)}{\left(V_{OC, u1}\right) + \left(\frac{KT_{u1}}{q}\right)} \right\} \right\} \left\{ \left\{ \left(I_{0, R1} \right) / \left(I_{SC, R1} + I_{0, R1} \right) \right\} \left\{ \frac{\left(\frac{\left(KT_{R1}}{q}\right) + \left(V_{OC, R1} \right) \right)}{\left(\frac{KT_{R1}}{q}\right)} \right\} \right\} \left\{ \left\{ \left(I_{0, u2} \right) / \left(I_{SC, u2} \right) + I_{0, u2} \right\} \right\} \left\{ \frac{\left(\frac{\left(KT_{u2}}{q}\right) + \left(V_{OC, u2} \right) \right)}{\left(\frac{KT_{u2}}{q}\right)} \right\} \right\} \left\{ \left\{ \left(I_{0, R2} \right) / \left(I_{SC, R2} + I_{0, R2} \right) \right\} \left\{ \frac{\left(\frac{\left(KT_{R2}}{q}\right) + \left(V_{OC, R2} \right) \right) \right\}}{\left(\frac{\left(KT_{R2}}{q}\right)} \right\} \right\} \dots \left\{ \left\{ \left(I_{0, uN} \right) / \left(I_{SC, RM} + I_{0, RM} \right) \right\} \left\{ \frac{\left(\frac{\left(KT_{RM}}{q}\right) + \left(V_{OC, RM} \right) \right) \right\}}{\left(\frac{\left(KT_{RM}}{q}\right)} \right)} \right\} \right\} \dots \left\{ \left\{ \left(I_{0, RM} \right) / \left(I_{SC, RM} + I_{0, RM} \right) \right\} \left\{ \frac{\left(\frac{\left(KT
$$

Simplifying eq.(38) by naming its R.H.S terms by " an electronic parameter constants ", such that:

$$
K_{u1} = \left\{ \left\{ \left(I_{SC,u1} + I_{0,u1} \right) / I_{0,u1} \right\} \left\{ \frac{\left(\frac{KT_{u1}}{q} \right)}{\left(V_{OC,u1} \right) + \left(\frac{KT_{u1}}{q} \right)} \right\} \right\}
$$
(39)

$$
K_{R1} = \left\{ \left\{ (I_{0,R1})/(I_{SC,R1} + I_{0,R1}) \right\} \left\{ \frac{\left[\left(\frac{KT,R1}{q} \right) + (V_{OC,R1}) \right]}{\left(\frac{KT,R1}{q} \right)} \right\} \right\}
$$
(40)

$$
K_{u2} = \left\{ \left\{ (I_{0,u2}) / (I_{SC,u2} + I_{0,u2}) \right\} \left\{ \frac{\left[\left(\frac{KT_{u2}}{q} \right) + (V_{OC,u2}) \right]}{\left(\frac{KT_{u2}}{q} \right)} \right\} \right\}
$$
(41)

$$
K_{R2} = \left\{ \left\{ (I_{0,R2}) / (I_{SC,R2} + I_{0,R2}) \right\} \left\{ \frac{\left[\left(\frac{KT_{R2}}{q} \right) + (V_{OC,R2}) \right]}{\left(\frac{KT_{R2}}{q} \right)} \right\} \right\}
$$
(42)

$$
K_{uN} = \left\{ \left((I_{0,uN})/(I_{SC,uN} + I_{0,uN}) \right) \left\{ \frac{\left[\left(\frac{KT_{uN}}{q} \right) + (V_{OC,uN}) \right]}{\left(\frac{KT_{uN}}{q} \right)} \right\} \right\}
$$
(43)

$$
K_{RM} = \left\{ \left\{ (I_{0,RM})/(I_{SC,RM} + I_{0,RM}) \right\} \left\{ \frac{\left[\left(\frac{KT_{RM}}{q} \right) + (V_{OC,RM}) \right]}{\left(\frac{KT_{RM}}{q} \right)} \right\} \right\}
$$
(44)

So that eq.(38) becomes:

$$
\frac{V_{m, u1}}{T_{u1}} - \frac{V_{m, R1}}{T_{u2}} - \frac{V_{m, R2}}{T_{u2}} - \dots - \frac{V_{m, uN}}{T_{uN}} - \dots - \frac{V_{m, RM}}{T_{RM}} =
$$
\n
$$
\left(\frac{k}{q}\right) \ln\{K_{u1} K_{R1} K_{u2} K_{R2} K_{u3} K_{R3} \dots K_{uN} \dots K_{RM}\}
$$
\nEq. (45) can be simplified to become:
\n
$$
\frac{V_{m, u1}}{V_{m, u1}} - \sum_{N} N_{v1} (V_{v1} - \langle T_{v1} \rangle) - \sum_{N} N_{v1} (V_{v1} - \langle T_{v1} \rangle) - \sum_{N} (V_{v1} - \langle T_{v1} \rangle)
$$

$$
\frac{V_{m, u1}}{T_{u1}} - \sum_{i=2}^{N} \{ (V_{m, u1} / T_{u1}) \} - \sum_{i=1}^{M} \{ (V_{m, Ri} / T_{Ri}) \} = {\frac{k}{q}} \{ (\prod_{i=1}^{N} K_{u1}) (\prod_{i=1}^{M} K_{Ri}) \}
$$
(46)

Any out of the N-unknown electric parameter pv cells (cell_{Ui}) its electrical parameter (here is the maximum voltage) can be obtained utilizing: single, some, or both the M-known electrical parameter pv cells (cell_{Ri}) for the determination of $V_{m,ui}$. for example, If the concerned pv cell (cell_{Ui}) = cell_{U1}, then the case of utilizing all the M- cell_{Ri} is derived when both sides of eq.(45) are multiplied by $T_{,U1}$, therefore eq.(45) becomes:

$$
v_{m,u1} - (T_{,u1}/T_{,R1})(V_{m,R1}) + (T_{,u1}/T_{,R2})(V_{m,R2}) + \cdots + (T_{,u1}/T_{,RM})(V_{m,RM}) \left(\frac{kT_{,u1}}{q}\right) \ln\{ [K_{u1}][\prod_{i=1}^{M} K_{Ri}] \}
$$
\n(47)

And hence, generally, the case of when single pv cell out of the N-cell_{Ui} is modeled with all the Mcell_{R,i} is then expressed as:

$$
\begin{bmatrix} V_{m,ui} \end{bmatrix} = \begin{bmatrix} T_{,ui} \end{bmatrix} \begin{bmatrix} V_{m,Ri} \end{bmatrix} + \begin{bmatrix} \frac{kT,ui}{q} \end{bmatrix} \begin{bmatrix} K_{ui} \end{bmatrix} \begin{bmatrix} \prod_{i=1}^{M} K_{Ri} \end{bmatrix}
$$
 (48)

Referring to eq(48), when, for example; $i = 1$ that means cell_{Ui} = cell_{U1} and hence expressed by: $[V_{m,u1}] = [T_{u1}][V_{m, Ri}] + (\frac{k}{2})$ $\left(\begin{matrix} \overline{u} \\ q \end{matrix}\right)$ { $\left[\begin{matrix} K_{ui1} \end{matrix}\right]$ $\left[\begin{matrix} \prod_{i=1}^{M} K_i \end{matrix}\right]$ (49)

Therefore, the general form of maximum voltage referring to any cell_{Ui} in terms of pv cells (cell_{Ri}) under any irradiance and thermal conditions is expressed by:

$$
V_{m,u} = \frac{\bar{T}_{,u}V_{m,R1}}{T_{,R1}} - \frac{T_{,u}V_{m,R2}}{T_{,R2}} - \frac{T_{,u}V_{m,R3}}{T_{,R3}} - \dots - \frac{T_{,u}V_{m,RM}}{T_{,RM}} \dots - \frac{T_{,u}V_{m,RN}}{T_{,RN}} =
$$
\n
$$
(50)
$$

Different optical irradiance and thermal conditions means different K_U and K_R . Consider the following pair of a single cell_U and a single cell_R, photovoltaically communicated with each other through this approach pv net model structure:

$$
V_{m,u} = \frac{T_{u}V_{m,R}}{T_{R}} + (kT_{vu}/q) \ln\{ \{ [\left(I_{SC,u} + I_{0,u} \right) / I_{0u}][(kT_{vu}/q) / ((kT_{,u}/q) + V_{oc,u})] \{ [I_{0,R} / I_{0u} + I_{0,R} \}][((kT_{R}/q) + V_{oc,R}) / (kT_{R}/q)] \} \} \tag{51}
$$

If cells (cell_U and cell_R) both are wide band gap pv cells, then, $V_{OC,u,R} >> kT_{u,R}/q$ and also $I_{SC,u,R} \gg I_{0,u,R}$ and as a result eq.(51) becomes:

$$
V_{m,u} = \frac{T_{,u}V_{m,R}}{T_{,R}} + (kT_{,u}/q) \ln\{ \{ [\,I_{SC,u} \, / I_{0,u}][T_{,u}/V_{oc,u} \,]] \} \{ [\,I_{0,R}/I_{SC,R} \,] [\, V_{oc,R} \, / T_{,R} \,] \} \} \tag{52}
$$
 Further expanding eq.(52), yields:

$$
V_{m,u} = \left\{ \frac{T_{,u}V_{m,R}}{T_{,R}} \right\} + \left\{ (kT_{,u}/q) \ln[\ I_{0,R}/I_{0,u}] \right\} + \left\{ (kT_{,u}/q) \ln[\ I_{\text{SC},u}/I_{\text{SC},R}][T_{,u}/T_{,R}][V_{\text{OC},R}/V_{\text{OC},u}] \right\} \tag{53}
$$

Substituting eq.(23), for the term $\left\{ (kT_{,u}/q) \ln[\ I_{0,R}/I_{0,u}] \right\}$, into eq.(53):

$$
V_{m,u} = \left\{ \frac{T_{,u}V_{m,R}}{T_{,R}} \right\} + \left\{ E_{g,u} - \left\{ E_{g,R}(T_{,u}/T_{,R}) \right\} \right\} + \left\{ (kT_{,u}/q) \ln\{[I_{SC,u}/I_{SC,R}][T_{,u}/T_{,R}][V_{OC,R}/V_{OC,u}]\} \right\}
$$
(54)

Therefore, eq.(54) is a relation of maximum voltages in terms of the band gap energies.

Referring to eqs.(24 through 27) at $T_{\text{u}} = T_{\text{R}}$, a derived equation of maximum voltage as a function of open circuit voltage concerning wide band gap pv cells (cell_{U_i} and cell_{R_i}) is obtained by substituting eqs.(24 through 27) into eq.(52), as follows:

$$
V_{oc,u} = (kT/q) \ln \left[I_{SC,u} / I_{0,u} \right], -V_{oc,R} = (kT/q) \ln \left[I_{0,R} / I_{SC,R} \right], \quad (kT/q) \ln \left[V_{oc,R} / V_{oc,u} \right] = (kT/q) \left\{ \ln(V_{oc,R}) - \ln(V_{oc,u}) \right\},
$$
 therefore
$$
eq(52)
$$
 becomes:

$$
V_{m,u} - V_{m,R} = \left\{ \left(\frac{kT_{,u}}{q} \right) \ln \left\{ \frac{I_{SC,u}}{I_{0,u}} \right\} \right\} + \left\{ \left(\frac{kT_{,u}}{q} \right) \ln \left\{ \frac{I_{0,R}}{I_{SC,R}} \right\} \right\} + \left\{ \left(\frac{kT_{,u}}{q} \right) \ln \left\{ \frac{V_{OC,R}}{V_{OC,u}} \right\} \right\} \tag{55}
$$

$$
V_{m,u} - V_{m,R} = \{V_{oc,u} - V_{oc,R}\} + \left\{ \left(\frac{kT_{,u}}{q}\right) \left\{ \ln(V_{OC,R}) - \ln(V_{OC,u}) \right\} \right\}
$$
 (56)

Equating eq.(54) and eq.(56) according to the term $V_{m,u} - V_{m,R}$ and by assuming the last R.H.S of eq.(54) negligible (\approx 0) as the case is valid for wide band gap pv cells, therefore:

$$
\left\{ E_{g,u} - \left\{ E_{g,R}(T_{,u}/T_{,R}) \right\} \right\} = \left\{ V_{oc,u} - V_{oc,R} \right\} + \left\{ \left(\frac{kT_{,u}}{q} \right) \left\{ \ln(V_{OC,R}) - \ln(V_{OC,u}) \right\} \right\}
$$
(57)

And hence:

$$
V_{oc,u} = V_{oc,R} + \{E_{g,u} - \{E_{g,R}(T_{u}/T_{,R})\}\} + \{(\frac{kT_{,u}}{q})\ln(V_{OC,R}/V_{OC,u})\}
$$
(58)

It is well examined that substituting $E_{g,R}$ for V_{OCR} and $E_{g,u}$ for $V_{OC,u}$, respectively, shows a very nearly same results obtained from eq.(58). Eq.(58) therefore can be written as:

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$$
\dots
$$
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 $V_{\text{o}c,u} = V_{\text{o}c,R} + \{E_{g,u} - \{E_{g,R}(T,u)/T,R)\}\} + \{(E_{g,h} - E_{g,h})\}$ $\frac{1}{q} \ln(E_{\rm g,R}/E_{\rm g,u})$ (59) Eq.(59) is really an important useful equation of finding out, in inverse way, the short circuit

current and hence the photon flux (N_{ph}), that is to say, Knowing only $E_{g,u}$ referring to cell_{Ui} and $E_{g,u}, E_{g,R}$, and V_{OCR} referred to cell_{Ri}, the open circuit voltage ($V_{OC,u}$) can be calculated as a result the light-generated current (the short circuit current) can easily also be calculated by substituting $V_{\text{OC},u}$ into eq.(24) where the reverse saturation current easily is obtained from eq.(4). From this calculated light-generated current therefore the photon flux (N_{ph}) can also be easily calculated using eq.(3).

Achieving equal maximum voltages of different wide band gap pv cells by temperature variation

 Regarding any pv cell parametric equation it is impossible, by direct substitution, to determine at which temperature does certain required maximum voltage is. Referring to the pv cell I-V characteristic equation shown below;

$$
I = I_0 \left(e^{(qV/kT)} - 1 \right) - I_{SC} \tag{60}
$$

It is clear, from eq.(60), that it is difficult to find out what the temperature (T) is at certain required voltage (V) and vice verse. This is because the pv current (I) that satisfies the condition is also unknown. Even if current (1) is known, the reverse saturation current $(1₀)$ is still a temperature dependent parameter as is clear from eq.(4).

Referring to eqs.(51, 52, and 23) for two certain pv cells of different wide band gap energy ($E_{g,u}$ and $E_{g,R}$), and since wide band gap energy pv cells makes possible to assume $V_{m,u} \gg kT_u/q$, and also $I_{SC,u} \gg I_{0,u}$, therefore:

$$
V_{m,u} = (T_{,u}/T_{,R})(V_{m,R}) + \{(E_{g,u}) - [(T_{,u}/T_{,R})(E_{g,R})]\} + (KT_{,u}/q) \ln\{[(V_{m,R}I_{0,R})/(I_{SC,u}T_u)/(V_{m,u}I_{0,u})]\}
$$
(61)

And for the widest band gap energy pv cell $(E_{g,u})$ required to drop its maximum voltage to be equal the maximum voltage of the less-wider pv cell ($E_{g,R}$); that is to say: $V_{m,u} = V_{m,R} = V_{m,T}$, where $V_{m,T}$ is the maximum voltage referred to an equal voltages $V_{m,u}$ and $V_{m,R}$.

$$
V_{m,T} = (T_{,u}/T_{,R})(V_{m,T}) + \{(E_{g,u}) - [(T_{,u}/T_{,R})(E_{g,R})]\} + (KT_{,u}/q) \ln\{[(V_{m,T})/(V_{m,T})] [((I_{SC,u})/(I_{SC,R}))] [(T_{,u}/T_{,R})/(V_{m,u}I_{0,u})]\}
$$
(62)

The last R.H.S term of equation (62) above is very small so that it can be neglected and hence eq.(62) can be approximated to:

$$
V_{m,T} = (T_{,u}/T_{,R})(V_{m,T}) + \{ (E_{g,u}) - [(T_{,u}/T_{,R})(E_{g,R})] \}
$$
\n(63)

$$
E_{g,u} - V_{m,T} = (T_{,u}/T_{,R}) \{ E_{g,R} - V_{m,T} \}
$$
\n(64)

Therefore, an excellent approximate equation defining the temperature required to make certain wider band gap pv cells maximum voltages equal is given by:

$$
T_{,u} = T_{,R}\{ (E_{g,u} - V_{m,T})/(E_{g,R} - V_{m,T}) \}
$$
\n(65)

Formulation of Maximum Current (Im)

In general, the maximum current (I_m) of pv cell is expressed by:

$$
I_{m} = \frac{\left(\frac{qV_{m}}{kT}\right)}{\left(\frac{qV_{m}}{kT}\right) + 1} \left[I_{SC} + I_{0}\right]
$$
\n
$$
(66)
$$

As a general relation, the maximum current of any cell_{Ui} can be written as:

$$
I_{m,ui} = \frac{\left(\frac{qV_{m,ui}}{KT_{,ui}}\right)}{\left(\frac{qV_{m,ui}}{KT_{,ui}}\right) + 1} \left(I_{SC,ui} + I_{0,i}\right) \tag{67}
$$

Where $V_{m,ui}$ is the unknown maximum voltage obtained referring to this approach analysis equations, eq.(48).

A new theoretical formulation of electrical parameters ……….……..Ahmed Saeed AL-Noban **Formulation of maximum power (Pm)**

Simply, the maximum power referring to this work approach is expressed as:

$$
P_{m,u} = I_{m,u} V_{m,u} = \left[\frac{\left(\frac{qV_{m,u}}{kT} \right)}{\left(\frac{qV_{m,u}}{kT} \right) + 1} [I_{SC,u} + I_{0,u}] \right] \left[(kT_{,u}/q) \ln\{ [K_u] [\prod_{i=1}^{M} [K_{Ri}] \} \right] \tag{68}
$$

As is the case concerned with wide band gap energy pv cells where $\left(\frac{q}{q}\right)$ $\frac{A \cdot m, u}{kT, u}$ >> 1 that leads automatically into $I_{SC,u}$ >> $I_{0,u}$, and hence eq.(68) becomes:

 $P_{m,u} \approx [I_{SC,u}][(kT_{,u}/q)ln\{ [K_u][\prod_{i=1}^{M}[K_i]] \}]$ $[3]$ (69)

Eq.(68) is a straight forward direct-substitution equation. With given data of K_u and K_R , the maximum power can directly be determined.

Results and discussions

 Referring to the given data (the incident sunlight power versus photon wavelengths) of air mass (AM1.5) sunlight spectrum radiation [2, 15], given in Table (1); Table (1) is completed by the conventional way of computations such that to include also all the electrical output parameters referring to a wide pv cell spectrum at working temperature and sunlight irradiance concentration ratio of 300 K, 1 sun, respectively as given by Table (2). The aim is to compare between electric parameters obtained conventionally in Table (2) and those referring to this approach formulation method. The conventional way of determining the electrical parameters, step by step is ordered as shown in Figure (1) as well as eqs. $(1 - 10)$.

 Generally, a very excellent matched results of electrical parameters obtained referring to this work approach (V_{mu} , $I_{m,u}$, and $P_{m,u}$) as compared to conventional electrical parameters (V_m , I_m , and P_m) given in Table (2) referring to conventional eqs.(1 - 10).

 Both, Table (3) and Figure (3) illustrate the case where several unknown electrical parameters pv cells (cell_{Ui}) ranging from 0.4887 ev to 3.815 ev each, utilizing either of the reference cells (cell_{Ri}): 0.4887 ev, 0.5838 ev, and 0.9288 ev both at same temperature $T_{u} = T_{R} = 300$ K and same concentration ratio $cr_u = cr_R = 1$ sun. As an example, cell_U of $E_{gu} = 1.2588$ ev is used to communicate with cells (cell_{Ri}) of E_{gR} = 0.4887 ev, 0.5838 ev, and 0.9288 ev leads to an excellent approximate results of maximum power (P_{mu}) of 20.5497 mW, 20.5967 mW, 20.4850 mW as compared with exact values obtained by the conventional way that results into maximum power (P_m) of 20.4456 mW, as an average of P_m / P_{mu} \times 100 % \approx 100 % is obtained. The pv-net model referring to this case is shown in figure (4).

Table (4), Figure (5), and Figure (6) both are deal with the case of the two pv cell groups (cell_{Ui} and cell_{Ri}) both operating under an equal concentration ratios of 1 sun but different temperature of T_{μ} = 400 K while T_{,R} = 300 K, respectively. Starting from cell_U = 0.6342 ev and ending to cell_U = 3.815 ev, the maximum power percentage ratios ($P_{m,u}/P_m$) × 100 % obtained are: 78%, 92.04 %, 92.5 %, 99.45 %, 99.485 %, 99.56 %, 99.6 %, 99.7015 %, 99.757 %, 99.7966 %, respectively.

Again Referring to Table (4) for cell_R of 0.4887 ev sharing several reference pv cells; cell_{Ui} = $[0.6342$ ev up to 3.815 ev], for the pair $[cell_U = 0.6342, cell_R = 0.4887$ ev], the electrical parameter percentage ratios of: $V_m = 87.25\%$ of $V_{m,u}$, $I_m = 90.3\%$ of $I_{m,u}$, and $P_m = 78.9\%$ of $P_{m,u}$ are obtained. On the other hand, when this narrow band gap pv cell of 0.6342 ev is replaced by a wide band gap pv cell (cell_u = 3.815 ev) such that the considered pair becomes [cell_U = 3.815 ev, cell_R = 0.4887 ev], the related electrical parameter percentage ratios are of: V_m = 99.8% of $V_{m,u}$, I_m = 99.998% of $I_{m,u}$, and P_m = 99.8% of $P_{m,u}$. In general, temperature increase affects both electrical parameters V_m , I_m , P_m , or, $V_{m,u}$, $I_{m,u}$, $P_{m,u}$. Comparing Tables (3 and 4) in terms of temperature variation, first when cell_U = 0.9288 ev that results into I_m = 40.689 mA, V_m = 0.375 v, and P_m = 15.275 mW at 300 K, while when temperature raised to 400 K, this results into $I_m = 37.477$ mA, V_m = 0.2144 v, and P_m = 8.037 mW. This drop in electrical parameter values at high temperature is mainly because of the increased value of the electrical parameter " I_0 " with temperature increase, especially when considering narrow band gap pv cells that results into a drop in V_{OC} and V_m because of the smaller temperature dependent ratio (qV_m/kT)/(qV_m/kT +1) that results into a drop in $I_{m,u}$ and $P_{m,u}$ also; second, wider band gap pv cells are less affected by temperature increase, as

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an example consider cell_U = 3.815 ev at $T = 300$ K, 400 K, results into electrical parameter values of: I_{mu} = 0.0201 mA, V_{mu} = 3.014 v, P_{mu} = 0.06 mW, and I_{mu} = 0.02006 mA, V_{mu} = 2.759 v, P_{mu} $= 0.0553$ mW, respectively. In short, referring to Table (4), starting from cell_{Ui} of 0.6342 ev and ending to cell_{Ui} of band gap 3.815 ev the maximum power percentage ratio P_m / P_{mu} × 100% is growing such as: 78%, 92.04%, 92.5%, 99.45%, 99.485%, 99.56%, 99.6%, 99.7015%, 99.757%, and 99.8%, respectively.

 The case that deal with the determination of electrical parameters with keeping unchanged the temperature of both pv cells (cell_{Ui} and cell_{Ri}) at 300 K, on the other hand, raising the concentration ratio of cell_{Ui} from 1 sun to 100 suns and keeping unchanged that of cell_{Ri} (i.e, at 1 sun) is shown in Table (5), Figure (7). Pv cells (cell_{Ui}): 0.4887 ev, 0.5838 ev, 1.81 ev, and 3.22 ev utilized with pv cells (cell_{Ri}) of: 0.4887 ev, 0.5838 ev, and 3.22 ev show ($P_{m,u}/P_m$) \times 100 % of 98.49 %, 99.875 %, 99.663 %; and, 98.892 %, 99.504 %, 99.546 %, and 99.743 %; and, 91.705 %, 96.762 %, 99.945 %, 99.9985 %, respectively. In general, concentrating sunlight increases both electrical parameters.

 The case concerned with varying both the temperature and the concentration ratio is illustrated by Figure (8) and Table (6). As a whole result average, about 99.9 % referring to $(V_{m,u}/V_m) \times 100\%$, $(I_{m,u}/I_m) \times 100\%$, and $(P_{m,u}/P_m) \times 100\%$ are obtained.

More than one cell_R when referred by (or photovoltaically net-communicated) with cell_U also show the good results the pair $\lceil \text{cell}_U, \text{cell}_R \rceil$ shows. This is illustrated in Table (7) and Figure (9) under T = 300 K and cr = 1 sun. From Table (7), it is clear that utilizing cell_{Ui} as wider band gap pv cell as possible as better are growing the results obtained.

Table (8) shows the variations of electrical parameter constants, K_U and K_R , of several band gap energy pv cells at T = 300 K and cr = 1 sun. As the band gap difference ($E_{g,R}$ - $E_{g,u}$) increases positively as the term $kT/q \ln \{ [K_U] [K_R] \}$ increases negatively. This term becomes zero only when $[K_U] = [K_R]$, provided that both considered pv cells (cell_U, cell_R) are operating under an identical irradiance and thermal conditions. Figure (10) shows the variations of V_{mu} with kT/q ln { $K_U K_R$ } for several pv cells when cell_R = 3.815 ev, V_{m,R} = 3.0088 v, at T_U = T_R = 300 K, cr_R = $cr_u = 1$ sun. It is obvious that this term $kT/q \ln {\{K_U K_R\}}$ decreases negatively until zero where cell_U = cell_R = 3.815 ev and the band gap difference ($E_{g,R}$ - $E_{g,u}$) = 0.

 Table (9) shows the calculated temperature concerning two different wide band gap pv cells such that their maximum voltages equal. An excellent average percentage ratio (checked exact T_u / approach approximate T_{μ}) of about 99.5%, (see eq. 65). Even the narrower band gap pv cells such as the pairs $[0.53448 \text{ ev}, 0.4887 \text{ ev}]$ and $[0.7626 \text{ ev}, 0.5838 \text{ ev}]$ give a comparable percentage results of 99.64%, 99.42%, respectively. Figure (11) shows the comparison between the approach and the conventional maximum current calculations at temperature 300 K and 400 K where cell_R of 0.4887 ev is used as a reference cell at $cr_R = 1 \text{ sun}, T_R = 300 \text{K}$. It is obvious how the approach results are very nearly coincide with the conventional results as all curves show and tables illustrate. Figure (12) shows the comparison between the approach and the conventional maximum voltage calculations for several cell_{Ui} at T_{ui} = 300 K, 400 K and cr_u = 1 sun, 100 suns where for all cases the reference cell is, cell_R = 0.4887 ev at T_R = 300 K, and cr_R = 1 sun. Figure (13) shows the variations of the maximum voltages for several pv cells under different T and cr using cells $(0.4887 \text{ ev}, 1.81 \text{ ev}, \text{and } 3.815 \text{ ev})$ each as a reference cell for each case. Figure (14) shows the comparison between the maximum voltages of conventional method, and approach method for several pv cells both at T =300 K, cr = 1 sun (i) cell_R = 0.4887 ev, (ii) cell_R = 0.5838 ev, (iii) cell_R = 0. 9288 ev, and (iv) cell_R = 3.815 ev. Figure (15) shows the comparison between the maximum power of the conventional method and the approach method for several pv cells, both at $T = 300$ K, cr = 1 sun for the following reference cells: (i) cell_R = 0.4887 ev, (ii) cell_R = 0.5838 ev, (iii) cell_R = 0. 9288 ev, and (iv) cell_R = 3.815 ev. Table (10) illustrates the calculated values of $V_{\text{OC,u}}$, $I_{SC,u}$, $N_{ph,u}(E)dE$ with only cell_{Ui} parameter known is the band gap $E_{g,u}$, using eq.(59).

A new theoretical formulation of electrical parameters ……….……..Ahmed Saeed AL-Noban Table (1): Air mass (AM1.5) sunlight radiation, sunlight incident power (P_{in}) versus

Table (2): Calculated electrical parameter of pv cells their band gap energy ranged within (0.4887

Table (3): Comparison between electrical parameters: (i) approach, cell_{Ui} = $[0.4887 \text{ eV}$ up to 3.815 ev], cell_{Ri} = [0.4887 ev, 0.5838 ev, and 0.9288 ev], cr_{Ri} = cr_{ui} = 1 sun and T_{,u} = T_{,R} = 300 K (ii) conventional, $T=300$ K, $cr = 1$ sun.

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\mathbf{I} TUV is, VI 1 ouil.							
CELL _R eV)	$CELL_{II}$ eV)	$V_{m,u}$, \mathbf{v})	V_m , (v)	$I_{m,u}$ (mA)	I_{m} (mA)	$P_{m,u}$ (mW)	P_m (mW)
0.4887	0.6342	0.0131122	0.01144	31.55653	28.5345	0.413777	0.32643
	0.6958	0.0347612	0.03675	31.13652	31.9988	1.082344	1.17595
	0.9288	0.2144199	0.21445	37.47725	37.4779	8.035870	8.03715
	.2588	0.5089647	0.50632	29.22966	29.2200	14.79491	14.8768
	1.5030	0.7336397	0.73002	23.20338	23.1982	17.02292	16.9351
	1.8100	1.0153607	1.01100	15.07690	15.0738	15.30758	15.2396
	2.1947	1.3679386	1.36305	7.859577	7.85888	10.75141	10.7120
	2.6600	1.7892424	1.78400	2.831431	2.83127	5.066118	5.05099
	3.2200	2.2817590	2.27630	0.504870	0.50485	1.151992	1.14919
	3.8150	2.7592394	2.75370	0.020064	0.02006	0.055363	0.05525

Table (4): Comparison between electrical parameters: (i) approach, cell_{Ui} = $[0.6342$ ev up to 3.815 ev], cell_{Ri} = 0.4887 ev, cr_u = cr_R = 1 sun, and T_{,u} = 400 K, T_{,R} = 300 K, (ii) conventional, $T = 400$ K, $cr = 1$ sun.

Table (5): Comparison between electrical parameters: (i) approach, cell_{Ui} = $[0.4887 \text{ ev}, 0.5838 \text{ ev},$ 1.81 ev, and 3.22 ev], cell_{Ri} = [0.4887 ev, 0.5838 ev, and 3.22 ev], cr_u = 100 suns, cr_R = 1 sun, and $T_{\text{u}} = T_{\text{Ri}} = 300 \text{ K}$. (ii) conventional, T = 300 K, cr = 100 suns.

CELL_R	- .uı CELL_{U}	- 771 $V_{m,u}$, (v)	V_m , (v)	$I_{m,u}$	I_{m}	$P_{m,u}$	P_m
$($ ev $)$	$($ ev $)$			(mA)	mA)	mW)	(mW)
0.4887	0.4887	0.090465	0.09160	4553.26	4565.832	411.913	418.2302
	0.5838	0.171720	0.17153	4754.26	4753.572	816.405	815.38034
	1.8100	1.320120	1.31575	1528.85	1528.759	2018.27	2011.4652
	3.2200	2.600000	2.61990	50.7501	50.74917	131.950	132.95774
0.5838	0.4887	0.091974	0.09160	4569.92	4565.832	420.314	418.23020
	0.5838	0.173228	0.17153	4759.68	4753.572	824.514	815.38034
	1.8100	1.321631	1.31573	1528.86	1528.758	2020.60	2011.4340
	3.2200	2.626533	2.61987	507.504	507.4916	1332.97	1329.5626
3.22	0.4887	0.085354	0.09160	4493.45	4565.83	383.536	418.2302
	0.5838	0.166609	0.17152	4734.44	4752.78	788.803	815.1963
	1.8100	1.315009	1.31572	1528.74	1528.75	2010.31	2011.418
	3.2200	2.619910	2.61987	50.7491	50.7491	132.958	132.9562

Table (6): Comparison between electrical parameters regarding (i) approach (ii) conventional way, for several randomly selected pv cells, temperature, and irradiance concentration ratio.

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a reference cells at $T_{\text{vii}} = T_{\text{rRi}} = 300 \text{ K}$ and $cr_{\text{Ui}} = cr_{\text{Ri}} = 1 \text{ sun}$, (ii) conventional.								
$CELL_{\rm Ri}$, ev	$\mathrm{CELL}_{\mathrm{U}}$, ev	V_{mu} , v	V_m , v	I_{mu} , mA	I_m , mA	P_{mu} , mW	P_m , mW	
$\begin{bmatrix} 0.9288, 1.503, 1.81, \end{bmatrix}$	0.4887	0.01856	0.01346	39.8965	32.6917	0.74082	0.4403	
2.1947, 3.815								
$\begin{bmatrix} 0.4887, 1.503, \end{bmatrix}$	0.9288	0.38257	0.37332	40.7391	40.6753	15.5858	15.185	
1.81 , 2.1947, 3.815]								
[0.4887, 0.9288, 1.81,	1.503	0.92323	0.91042	23.6321	23.6231	21.8179	21.507	
2.1947, 3.815								
[0.4887, 0.9288, 1.503]	1.81	1.21265	1.19900	15.2625	15.2589	18.5081	18.295	
, 2.1947, 3.815								
$\begin{bmatrix} 0.4887, 0.9288, 1.503 \end{bmatrix}$	2.1947	1.57430	1.55997	7.927510	7.92632	12.4803	12.364	
$, 1.81, 3.815$]								
$\begin{bmatrix} 0.4887, 0.9288, 1.503 \end{bmatrix}$	3.815	3.02443	3.00880	0.02014	0.02014	0.06092	0.0606	
, 1.81, 2.1947								

Table (7): Comparison between electrical parameters (i) approach, for five cells used as

Table (8): Variations of electrical parameter constants, K_U and K_R , of several pv cells band gap energy at $T = 300$ K and $cr = 1$ sun.

A system of pv cells, (ev)	K_{U}	K_{R}
0.4887	1.319738831	0.757725677
0.9288	1584991.487	6.30918×10^{-7}
1.5030	1.7855×10^{15}	5.6004×10^{-16}
1.8100	1.2771×10^{20}	7.8300×10^{-21}
2.1947	1.4970×10^{26}	6.6796×10^{-27}
3.8150	3.3325×10^{50}	3.0007×10^{-51}

Table (9): Calculated temperature concerning a pair of different wide band gap pv cells such that their maximum voltage equal.

$[CELL_U, CELL_R]$ ev	$V_{m,u}(V)$	$V_{m,R}$, (V)	T_R , (K)	$T_u(K)$, Approx. equation	$T_u(K)$, checked exact equation
2.1947, 1.81	1.1990085	1.1990085	300	488.88969	484.969
[0.7626, 0.5838]	0.0712420	0.0712420	300	404.65157	407.01
$1.8646, 1.1262$]	0.5563366	0.5563366	300	688.72400	683.5
[3.220, 2.660]	1.9924891	19924891	300	578.64713	575.12
[0.53448, 0.4887]	0.0134685	0.0134685	300	328.88960	330.0615

Table (10): Calculations of $V_{OC,u}$, $I_{SC,u}$, $\int N_{ph,u}(E)dE$ with only known parameter is the band gap $E_{\text{g},u}$, at T_{,u} = T_{,R}, cr_u = cr_R, using eq.(59).

Figure (1): Calculations of electrical parameters of pv cell according to their priority, starting from the input sunlight power (P_{in} **) and ending to the conversion efficiency (** η **)**

Figure (2): The approach general pv cells net- structure of both groups (N-cell_{Ui} and M **cellRi)**

Figure (3): Comparison between maximum power variation for several pv cells (i) approach, $T_{\text{ini}} = T_{\text{.}Ri} = 300 \text{ K}$, $\text{cr}_u = \text{cr}_R = 1 \text{ sun}$, $\text{cr}_u = 1 \text{ sun}$, (ii) conventional, T = 300 K, cr **= 1 sun.**

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0.6342 ev 0.6958 ev 0.9288 ev 1.2588 ev 1.503 ev 1.81 ev 2.1947 ev 2.66 ev 3.22 ev 3.815 ev 0.4887 ev

Figure (4): Cell_{Ui} and Cell_{Ri} net- structure, Figure (5): Cell_{Ui} and Cell_{Ri} net-structur, $T_{yui} = T_{yki} = 300 \text{ K}, \text{ } \text{cr}_u = \text{cr}_R = 1 \text{ sun}.$
T_{1ui} = 400 K, $T_{yki} = 300 \text{ K}, \text{ } \text{cr}_u = \text{cr}_R = 1 \text{ sun}$

Figure (6): Comparison between maximum power variation for several pv cells (i) approach, cell_{Ri} = 0.4887 ev, T_{,u} = 400 K, T_{,R} = 300 K, cr_u = cr_R = 1 sun, (ii) conventional, **T=400 K, cr = 1 sun.**

Figure (10): Variations of $V_{m,u}$ **with kT/q ln{** $K_U K_R$ **} for several pv cells when cell_R = 3.815 ev,** $V_{m,R} = 3.0088$ **v,** at $T_{v} = T_{rR} = 300$ **K,** $cr_{U} = cr_{R} = 1$ sun.

Figure (11): Comparison between the approach and conventional maximum current calculations for several cell_{Ui} at $T_{ui} = 300$ K, 400 K and $cr_u = cr_R = 1$ sun, where for all cases the reference cell used is, $\text{cell}_R = 0.4887$ ev at $T_{R} = 300$ K, and $\text{cr}_R = 1$ sun.

Figure (12): Comparison between the approach and conventional maximum voltage calculations for several cell_{Ui} at $T_{ui} = 300$ K, 400 K and cr_u = 1 sun, 100 suns, where for all **cases the reference cell used is, cellR** $= 0.4887$ ev at $T_R = 300$ K, and $cr_R = 1$ sun.

Figure (13): The variations of maximum voltages for several pv cells under different T and cr using cells (0.4887 ev, 1.81 ev, and 3.815 ev) each as a reference cell for each case.

Figure (14): Comparison between maximum voltages of conventional method, and approach method for several pv cells both at T = 300 K, $cr = 1 \text{ sun (i) cell}_R = 0.4887 \text{ ev}$ **,** (ii) cell_R = 0.5838 ev, (iii) cell_R = 0.9288 ev, and (iv) cell_R = 3.815 ev.

CONCLUSIONS

1- From Table (2) that concerned with the conventional calculations of electrical parameters, at fixed temperature and sunlight irradiance, it is concluded that: the wider are the band gap (E_g) of the utilized pv cells the smaller are their electrical parameters I_0 and $I_{\rm SC}$, and the larger are their electrical parameters; V_{OC} , FF, V.F, V_m , and I_m because of their larger related ratios (qV_m/kT) /{(1 + (qV_m/kT) } and {(I_{SC} / I₀) + 1}. Both the maximum power (P_m) and the conversion efficiency (η) increase starting from the narrowest band gap pv cell until the largest maximum power point $(I_m V_m)$ referred to the highest efficient pv cell is reached beyond which P_m and η drop. Regarding this approach analysis and formulations, electrical parameters obtained show a satisfied matched results as compared to the way these electrical parameters are conventionally calculated given in Table (2).

Very narrower pv cells, those, in fact, are theoretically only exist such as the pv cell of referred band gap 0.4887 ev, show a deviation only at high temperatures. This is because the temperature dependent electrical parameter " I_0 " increases with temperature increase making the two ratios, $(qV_m /kT) / \{(1 + (qV_m /kT))\}$ and $\{(I_{SC}/I_0) + 1\}$, meant by calculation of both V_{OC} , V_m and I_m , being a smaller ratios. Wider band gap pv cells have (q V_m/kT) >> 1 and, as a result, both their electrical output parameters are less affected by temperature increase.

- 2- A simple program, regardless of any of the interpolation techniques, can easily be designed for the computation of all electrical parameters under any irradiance and temperature variation conditions by only inputting the electrical parameter constants K_{ui} and K_{Ri} .
- 3- Regarding eq.(65) as a very important equation, determining the required increment in temperature regarding certain wide band gap energy pv cell so that its voltage falls down to equal certain maximum voltage referred to certain slightly smaller band gap energy pv solar cell. This in fact may comes to be an important issue in future when different types and different area pv cells have to be connected either in series or parallel or mixed connection (serial-parallel connection). Many pairs of suitable photovoltaic cells may look for in order to support this issue.
- 4- By inverse way, calculating the open circuit voltage first before calculating the lightgenerated current is possible regarding this approach by only substituting E_{gu} for $V_{OC,u}$ as given by the approximate equation (eq. 59) and, as a result, the photon flux is easily then obtained by substituting into eq.(3). This is achieved by knowing no other parameter than the band gap of the considered pv cell referred to the known electrical parameter V_{OCR} and $E_{g,R}$ of pv cell_R as given by eqs.(58 and 59). A new band gap pv cells not covered by Table (2) can be added, as given in Table (10).
- 5- As wider is $E_{g,u}$ as larger is its K_u . $\{[K_u] | [K_R] \} \neq 1$ at different T and different cr when same pv cell plays, at the same time, both as cell_U as well as cell_R. Under identical thermal and optical conditions, utilizing the same pv cell to acts as cell_U and cell_R yields into: $[K_u][K_R] = 1$, and therefore; $K T/q \ln \{[K_u][K_u]\} = 0$. On the other, hand for different thermal and optical conditions, this case can be stated as: $[K_u] [K_R] \neq 1$, and therefore, kT/q ln ${K_u | K_{u} | K_R} \neq 0$. $K_u \neq K_R$ means, either (i) the pv cell pair used (cell_U, cell_R) is non- identical type pv cell pair regardless of any irradiance or thermal conditions, or (ii) the pv cell pair used (cell_U, cell_R) is an identical type pv cell pair, but each of its pv cells are
- operating under different irradiance and thermal condition from the other. 6- This work idea can be generalized to any solar spectrum radiation other than the utilized one (AM1.5).

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صياغة نظرية جديدة لعوامل اخلرج الكهربائية ذات العالقة بقدرة اخلاليا

الفوتوفولطائية العظمى

أحمد سعيد النوبان

قسم الهندسة الإلكترونية والاتصالات, كلية الهندسة, جامعة عدن, عدن, اليمن. DOI: <https://doi.org/10.47372/uajnas.2016.n1.a11>

الملخص

يقدم هذا البحث صبياغة نظرية تحليلية جديدة لعوامل الخرج الكهربائية تنتهي باشتقاق معادلة مبسطة لقدرة الخرج العظمى في الخلايا الفوتوفولطائية. يركز هذا البحث بصورة اساسية على العوامل الكهربائية الهامة التي لا يمكن صياغتها صراحة بدلالة المتغيرات الكهربائية الأخرى كالفولتية العظمي والتيار الأعظم وبالتالي القدّرة العظمى للخلية الفوتوفولطائية. يستخدم هذا البحث ثابت شمسي ذو كتلة هواء (1.5). بداية: وفقا للقدرة الساقطة-الأطوال الموجية المتوفرة من خلال طُبِف كتلة الهواء هذا حَم ، تقليديا ، حساب جميع العوامل الكهربائية لطيف واسع من الخلايا الفوتوفولطائية بِتراوح بين 0.4887 - 3.815 إلكترون فولت لغرض المقارنة ببِنها وبين القيم المناط بها أسلوب هذا البحث. ثانيا: بعيدا عن النمذجة التقليدية للخلايا الفوتوفولطائية الذي يعتمد استقلالية كل خلية بعينها في نمذجة المتغيرات الكهربائية ينهج هذا البحث اسلوبا مغايرًا حيث يتم نمذجة الخلايا الفوتوفولطائية للمتغير الكهربائي المجهول، اعتمادياً غير استقلالياً، بدلالة العديد من الخلايا الفوتو فولطائية المعلومة المتغير "ذاته. خلاصية هذا البحث تمخض عن اشتقاق العديد من العلاقات المتحققة حتّى نحو بر في إلى100 %، دون الخوض في تعقيدات الطرق والتقنيات التقليدية.

ا**لكلمات المفتاحية**: الخلابا الفو تو فو لطائبة، عو امل الخلابا الفو تو فو لطائبة الكهر بائبة ِ