Different types of decomposition for certain tensors in K^h-BR-F_n and K^h-BR- affinely connected space

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Abstract

In this paper we defined K^h-birecurrent space which is characterized by the condition $K^i_{jkh|m|\ell} = a_{\ell m} K^i_{jkh}$, $K^i_{jkh} \neq 0$, also we introduced some decompositions of Cartan's fourth and third curvature tensor and Berwald curvature tensor and its torsion tensor.
The aim of this paper is devoted to the discussion of decomposition for different tensors in K^h-birecurrent space and K^h-birecurrent affinely connected space and the decomposition of curvature tensor Cartan's fourth and third in K^h-birecurrent space, also the decomposition of curvature tensor of Berwald in K^h-birecurrent affinely connected space, various results, formulas, theorems and different identities have been obtained.

Key words: Decomposition of curvature tensor, K^h-BR-affinely connected decomposition of Cartan's fourth curvature tensor and decomposition of Berwald curvature tensor.

Introduction

The decomposition of curvature tensor of recurrent manifold was discussed first by K. Takano [14], B. B. Sinha and S. P. Singh [12], B. B. Sinha and G. Singh [13] and others. R. Hit [9] introduced a recurrent Finsler space whose Berwald curvature tensor is decomposition in the form $H^i_{jkh} = X^i \Upsilon_{jkh}$ and obtained several results. H. D. Pande and H. S. Shukla [3] discussed the decomposition of curvature tensor filed K^i_{jkh} and H^i_{jkh} in recurrent Finsler space and studies properties of such decomposition. H. D. Pande and T. A. Khan[2]consider a recurrent Finsler space whose Berwald curvature tensor is decomposition in the form $H^i_{jkh} = X^i_j \Upsilon_{kh}$, while another decomposition of the form $H^i_{jkh} = P_j X^i_{hk}$ was proposed by H. D. Pande and H. S. Shukla [3]. P. N. Pandey [4] discussed the problem of decomposition of curvature tensor of a Finsler manifold restricting himself to Berwald curvature tensor H^i_{jkh} , P. N. Pandey [5] discussed the decomposition of curvature tensor of É. Cartan. B. B. Sinha [11] studies birecurrent Finsler space whose Berwald curvature tensor is decomposition in the form $H^i_{jkh} = Y^i \Upsilon_{jkh}$.

Let F_n be an n-dimensional Finsler space required with the metric function $F(x, y)$ satisfies the requisite condition [10].

É. Cartan [10] deduced the h-covariant differentiation for an arbitrary vector field X^i with respect to x^k as follows:

(1.1) $X^i_{|k} \coloneqq \partial_k X^i + X^r \Gamma^{*i}_{rk} - (\dot{\partial}_r X^i) G^r_k$.

The vector y^i vanish under h-covariant differentiation, i.e.

(1.2) $y^i_{|k} = 0$.

Due to homogenous of Γ^{*i}_{jk} in y^i the connection parameter Γ^{*i}_{jk} satisfies [10]

(1.3) $(\dot{\partial}_h \Gamma^{*i}_{jk}) \, y^h = 0$.

The commutation formula for h-covariant differentiation of an arbitrary vector field X^i is given by Rund[10]

(1.4) $X^i_{|k|j} - X^i_{|j|k} = X^r K^i_{rkj} - (\dot{\partial}_r X^i) K^r_{skj} y^s$.

The tensor K^i_{jkh} is called *Cartan's fourth curvature tensor* which is skew-symmetry in it's last two lower indicts k and j, i.e.

(1.5) $K^i_{jkh} = -K^i_{jhk}$

and satisfy the following identities known as *Bianchi identities*.

(1.6) a) $K^i_{jkh} + K^i_{hjk} + K^i_{khj} = 0$

and

(1.7) b) $K^r_{ijh|k} + K^r_{ikj|h} + K^r_{ihk|j} + (\dot{\partial}_s \Gamma^{*r}_{ij}) K^s_{mhk}\, y^m$
$+ (\dot{\partial}_s \Gamma^{*r}_{ik}) K^s_{mjh} y^m + (\dot{\partial}_s \Gamma^{*r}_{ih}) K^s_{mkj}\, y^m = 0$.

The curvature tensor K^i_{jkh} satisfy the following relation too

(1.8) $K^i_{jkh}\, y^j = H^i_{kh}$

and

(1.9) $H^i_{mkh} - K^i_{mkh} = P^i_{mk|h} + P^r_{mk}\, P^i_{rh} - k/h$.

The associate curvature tensor K_{ijkh} of the curvature tensor K^r_{jkh} is given by

(1.10) $K_{ijkh} := g_{rj}\, K^r_{jkh}$.

The tensor R^i_{jkh} is called *Cartan's third curvature tensor* satisfied the relation

(1.11) $R^i_{jkh} = K^i_{jkh} + C^i_{jm}\, H^m_{kh}$.

The curvature tensor R^i_{jkh} satisfies the following identity known as *Bianchi identity.*

(1.12) $R^i_{jkh|s} + R^i_{jsk|h} + R^i_{jhs|k}$
$+ (R^r_{mhs}\, P^i_{jkr} + R^r_{mkh}\, P^i_{jsr} + R^r_{msk}\, P^i_{jhr})\, y^m = 0$,

where P^i_{jkr} is called h-curvature tensor (*Cartan's second curvature tensor*) satisfies the relation

(1.13) $P^i_{jkh} y^j = \Gamma^{*i}_{jkh} y^j = P^i_{kh} = C^i_{kh|r} y^r$,

where

$$P^i_{kh} = (\dot{\partial}_k \Gamma^{*i}_{jh}) y^j = (\dot{\partial}_k \Gamma^{*i}_{hj}) y^j \quad .$$

The associate curvature tensor R_{ijkh} of the curvature tensor R^i_{jkh} is given by

(1.14) $R_{ijkh} = g_{rj}\, R^r_{ikh}$.

Berwald curvature tensor H^i_{rkh} and the h(v)-torsion tensor H^i_{kh} are related by

(1.15) $H^i_{rkh}\, y^r = H^i_{kh}$

and

(1.16) $H^i_{rkh} = \dot{\partial}_r H^i_{kh}$.

The associate curvature tensor H_{ijkh} of the curvature tensor H^i_{jkh} is given by

(1.17) $H_{ijkh} = g_{rj}\, H^r_{ikh}$.

The deviation tensor H^i_k satisfies the following:

(1.18) $g_{ij}\, H^i_k = g_{ik}\, H^i_j$.

Definition 1.1. A Finsler space whose connection parameter G^i_{jk} is independent of the direction argument y^i is called an *affinely connected space* (Berwald space). Thus, an affinely connected space characterized by any one of the following condition

(1.19) a) $G^i_{jkh} = 0$ and b) $G_{ijk|h} = 0$.

The connection parameter Γ^{*i}_{jk} of Cartan and G^i_{jk} of Berwald coincide in affinely connected space and they are independent of the direction argument [10]

(1.20) a) $G^i_{jkh} = \dot{\partial}_j G^i_{kh} = 0$ and b) $\dot{\partial}_j \Gamma^{*i}_{hk} = 0$.

A h-Birecurrent Tensor

A Finsler space for which Cartan's fourth curvature tensor K^i_{jkh} satisfies the birecurrent property with respect to Cartan's connection parameter Γ^{*i}_{jk} is called K^h*-birecurrent space*. Thus, K^h-birecurrent space is characterized by condition

(2.1) $K^i_{jkh|m|\ell} = a_{\ell m} K^i_{jkh}$, $K^i_{jkh} \neq 0$, { (2.2), [1]; (2.1), [7]; (2.1), [8] },

where the non-zero covariant tensor field of second order $a_{\ell m}$ being recurrence tensor field. The tensor satisfies the condition (2.1) is called *h-birecurrent tensor*. Such space and tensor denoted briefly by K^h-*BR*-F_n and *h-BR* , respectively.

Let us consider K^h-*BR*-F_n which is characterized by the condition (2.1).

Transvecting the condition (2.1) by y^j, using (1.2) and (1.8), we get

(2.2) $H^i_{kh|m|\ell} = a_{\ell m} H^i_{kh}$.

If we interchange the indices m and ℓ in the condition (2.1) and subtracting the equation obtained from the condition (2.1), we get

(2.3) $K^i_{jkh|m|\ell} - K^i_{jkh|\ell|m} = (a_{\ell m} - a_{m\ell})\, K^i_{jkh}$.

Definition 2.1. The K^h-birecurrent space which is affinely connected space [satisfy any one of the conditions (1.19a), (1.19b) or (1.20b)] will be called K^h*-birecurrent affinely connected space*. We shall donate it briefly by K^h*-BR- affinety connected space*.

Decomposition of Some Tensors in $\mathbf{K^h}$-BR-$\mathbf{F_n}$

H. D. Pande and H. S. Shukla [3] discussed the decomposition of the curvature tensor filed K^i_{jkh} in K^h-recurrent space. Thus, the decomposition of Cartan's fourth curvature tensor K^i_{jkh} is characterized by

(3.1) $K^i_{jkh} = y^i \Psi_{jkh}$,

where Ψ_{jkh} is a non-zero homogenous tensor of degree -1 in its directional argument is called *decomposition tensor filed* and

(3.2) $y^i \mathrm{V}_i = \sigma$.

In view of (3.1), the identities (1.5) and (1.6) can be written as

(3.3) $\Psi_{jkh} + \Psi_{jhk} = 0$

and

(3.4) $\Psi_{jkh} + \Psi_{hjk} + \Psi_{khj} = 0$,

respectively.

Let us consider K^h-*BR*-F_n which is characterized by the condition (2.1).

Taking the h-covariant derivative for (3.1) with respect to x^m and using (1.2), we get

(3.5) $K^i_{jkh|m} = y^i \Psi_{jkh|m}$.

Taking the h-covariant derivative for (3.5) with respect to x^l , using (1.2) and the condition (2.1), we get

(3.6) $a_{lm} K^i_{jkh} = y^i \Psi_{jkh|m|l}$.

Putting (3.1) in (3.6), we get

(3.7) $\Psi_{jkh|m|l} = a_{lm} \Psi_{jkh|m|l}$,

since $y^i \neq 0$.

Thus, we conclude

Theorem 3.1. *In* K^h*-BR-*F_n*, the decomposable tensor filed* Ψ_{jkh} *behaves as* $h - BR$.

If we interchange the indices m and l in (3.7) and subtracting the equation obtained from (3.7), we get

(3.8) $\Psi_{jkh|m|l} - \Psi_{jkh|l|m} = (a_{lm} - a_{ml})\, \Psi_{jkh}$.

Using the commotion formula exhibited by (1.4) in (3.8), we get

(3.9) $(a_{lm}-a_{ml})\,\Psi_{jkh}=-\left(\Psi_{rkh}K^r_{jlm}+\Psi_{jrh}K^r_{klm}+\Psi_{jkr}K^r_{hlm}-\dot{\partial}_r\Psi_{jkh}K^r_{slm}y^s\right).$

Using (3.1) and the homogeneity property of Ψ_{jkh} in (3.9), we get

(3.10) $(a_{lm}-a_{ml})\,\Psi_{jkh}=-\left(\Psi_{rkh}\Psi_{jlm}+\Psi_{jrh}\Psi_{klm}+\Psi_{jkr}\Psi_{hlm}-\Psi_{jkh}\Psi_{rlm}\right)y^r.$

Taking the h-covariant derivative for (3.10), twice with respect to x^n and x^p, successively and using (1.2), we get

(3.11) $(a_{lm}-a_{ml})_{|n|p}\Psi_{jkh}+(a_{lm}-a_{ml})_{|n}\Psi_{jkh|p}+(a_{lm}-a_{ml})_{|p}\Psi_{jkh|n}$
$+(a_{lm}-a_{ml})\Psi_{jkh|n|p}=-(\Psi_{rkh|n|p}\Psi_{jlm}+\Psi_{rkh|n}\Psi_{jlm|p}+\Psi_{rkh|p}\Psi_{jlm|n}$
$+\Psi_{rkh}\Psi_{jlm|n|p}+\Psi_{jrh|n|p}\Psi_{klm}+\Psi_{jrh|n}\Psi_{klm|p}+\Psi_{jrh|p}\Psi_{klm|n}+\Psi_{jrh}\Psi_{klm|n|p}$
$+\Psi_{jkr|n|p}\Psi_{hlm}+\Psi_{jkr|n}\Psi_{hlm|p}+\Psi_{jkr|p}\Psi_{hlm|n}+\Psi_{jkr}\Psi_{hlm|n|p}-\Psi_{jkh|n|p}\Psi_{rlm}$
$-\Psi_{jkh|n}\Psi_{rlm|p}-\Psi_{jkh|p}\Psi_{rlm|n}-\Psi_{jkh}\Psi_{rlm|n|p})\,y^r.$

Using (3.1) and (3.10) in (3.11), we get

(3.12) $(a_{lm}-a_{ml})_{|n|p}\Psi_{jkh}+(a_{lm}-a_{ml})_{|n}\Psi_{jkh|p}+(a_{lm}-a_{ml})_{|p}\Psi_{jkh|n}$
$=-b_{pn}\left(\Psi_{rkh}\Psi_{jlm}+\Psi_{jrh}\Psi_{klm}+\Psi_{jkr}\Psi_{hlm}-\Psi_{jkh}\Psi_{rlm}\right)y^r$
$-(\Psi_{rkh|n}\Psi_{jlm|p}+\Psi_{rkh|p}\Psi_{jlm|n}+\Psi_{jrh|n}\Psi_{klm|p}+\Psi_{jrh|p}\Psi_{klm|n}$
$+\Psi_{jkr|n}\Psi_{hlm|p}+\Psi_{jkr|p}\Psi_{hlm|n}-\Psi_{jkh|n}\Psi_{rlm|p}-\Psi_{jkh|p}\Psi_{rlm|n})\,y^r$.

Again, using (3.10) in (3.12), we get

(3.13) $(a_{lm}-a_{ml})_{|n|p}\Psi_{jkh}+(a_{lm}-a_{ml})_{|n}\Psi_{jkh|p}+(a_{lm}-a_{ml})_{|p}\Psi_{jkh|n}$
$=-b_{pn}(a_{lm}-a_{ml})\Psi_{jkh}-(\Psi_{rkh|n}\Psi_{jlm|p}+\Psi_{rkh|p}\Psi_{jlm|n}+\Psi_{jrh|n}\Psi_{klm|p}$

$+\Psi_{jrh|p}\Psi_{klm|n}+\Psi_{jkr|n}\Psi_{hlm|p}+\Psi_{jkr|p}\Psi_{hlm|n}-\Psi_{jkh|n}\Psi_{rlm|p}-\Psi_{jkh|p}\Psi_{rlm|n})y^r$

which can be written as

(3.14) $(a_{lm}-a_{ml})_{|n|p}=-b_{pn}(a_{lm}-a_{ml})$

if and only if

(3.15) $(a_{lm}-a_{ml})_{|n}\Psi_{jkh|p}+(a_{lm}-a_{ml})_{|p}\Psi_{jkh|n}=-(\Psi_{rkh|n}\Psi_{jlm|p}$
$+\Psi_{rkh|p}\Psi_{jlm|n}+\Psi_{jrh|n}\Psi_{klm|p}+\Psi_{jrh|p}\Psi_{klm|n}+\Psi_{jkr|n}\Psi_{hlm|p}$
$+\Psi_{jkr|p}\Psi_{hlm|n}-\Psi_{jkh|n}\Psi_{rlm|p}-\Psi_{jkh|p}\Psi_{rlm|n})\,y^r$,

since $\Psi_{jkh}\neq 0$.

Thus, we conclude

Theorem 3.2. *In K^h-BR-F_n, the tensor $(a_{lm}-a_{ml})$ behaves as $h-BR$ under the decomposition (3.1) if and only if (3.15) holds good.*

Using (3.1) in the identity (1.7), we get

(3.16) $\left(\Psi_{jkh|m}+\Psi_{jmk|h}+\Psi_{jhm|k}\right)y^i+\{\left(\dot{\partial}_s\Gamma^{*i}_{jk}\right)\Psi_{rhm}+\left(\dot{\partial}_s\Gamma^{*i}_{jm}\right)\Psi_{rkh}$
$+\left(\dot{\partial}_s\Gamma^{*i}_{jh}\right)\Psi_{rmk}\}\,y^sy^r=0$.

Using (1.3) in (3.16), we get

(3.17) $\left(\Psi_{jkh|m}+\Psi_{jmk|h}+\Psi_{jhm|k}\right)y^i=0$

Taking the h-covariant derivative for (3.17), with respect to x^l, using (1.2) and (3.7), we get

(3.18) $\left(\Psi_{jkh|m|e}+\Psi_{jmk|h|e}+\Psi_{jhm|k|e}\right)y^i=0$,

which can be written as

(3.19) $a_{lm}\Psi_{jkh}y^i=\left(-a_{lh}\Psi_{jmk}-a_{lk}\Psi_{jhm}\right)y^i$.

Using (3.3) in (3.19), we get

(3.20) $a_{lm}\Psi_{jkh}y^i=\left(a_{lh}\Psi_{jkm}+a_{lk}\Psi_{jmh}\right)y^i$.

Using (3.1) in (3.20), we get

$$a_{lm}\, K^i_{jkh} = (a_{lh}\, \Psi_{jkm} + a_{lk}\Psi_{jmh})y^i$$

or

(3.21) $K^i_{jkh} = \frac{1}{a_{lm}}(a_{lh}\, \Psi_{jkm} + a_{lk}\Psi_{jmh})y^i$.

Thus, we conclude

Theorem 3.3. *In K^h-BR-F_n, Cartan's fourth curvature tensor K^i_{jkh} is defined by (3.21) under the decomposition (3.1).*

Further considering the decomposition of the tensor filed Ψ_{jkh} in the form

(3.22) $\Psi_{jkh} = \nu_j \Psi_{kh}$,

where Ψ_{kh} is non-zero tensor filed [3].

Taking the h-covariant derivative for (3.22) with respect to x^m, we get

(3.23) $\Psi_{jkh|m} = \nu_{j|m}\Psi_{kh} + \nu_j \Psi_{kh|m}$.

Taking the h-covariant derivative for (3.23) with respect to x^l and using (3.7), we get

(3.24) $a_{lm}\Psi_{jkh} = \nu_{j|m|l}\Psi_{kh} + \nu_{j|m}\Psi_{kh|l} + \nu_{j|l}\Psi_{kh|m} + \nu_j \Psi_{kh|m|l}$.

Putting (3.22) in (3.24), we get

(3.25) $a_{lm}\nu_j\Psi_{kh} = \nu_{j|m|l}\Psi_{kh} + \nu_{j|m}\Psi_{kh|l} + \nu_{j|l}\Psi_{kh|m} + \nu_j \Psi_{kh|m|l}$.

Transvecting (3.25) by y^j , using (3.2) and (1.2), we get

(3.26) $a_{lm}\sigma\Psi_{kh} = \sigma_{|m|l}\Psi_{kh} + \sigma_{|m}\Psi_{kh|l} + \sigma_{|l}\Psi_{kh|m} + \sigma\, \Psi_{kh|m|l}$.

If σ is constant, (3.26) can be written as

(3.27) $\Psi_{kh|m|l} = a_{lm}\Psi_{kh}$,

since $\sigma_{|m} = 0$.

Thus, we conclude

Theorem 3.4. *In K^h-BR-F_n, the tensor filed Ψ_{kh} behaves as $h - BR$ under the decompositions (3.1) and (3.22) provided that σ is constant.*

Decomposition of Some Tensors in $\mathrm{K^h}$-BR- Affinely Connected Space

Let us consider a $\mathrm{K^h}$-BR- Affinely connected space.

Transvecting the equation $a_{lk}\, K^i_{jhm} - a_{hm}\, K^i_{jlk} = 0$, {(3.9), [8]}

by the non-zero contravariant tensor z^{hm} such that $a_{hm}\, z^{hm} = \phi \neq 0$ (non-zero scalar, i.e. tensor of zero order), we get

(4.1) $\phi\, K^i_{jlk} = a_{lk}\, K^i_{jhm}\, z^{hm}$.

In view of quotient low, the equation (4.1) can be written as

(4.2) $K^i_{jlk} = a_{lk}\, X^i_j$,

where $X^i_j = \frac{K^i_{jlk}\, z^{hm}}{\phi}$, since $K^i_{jlk}\, z^{hm} \neq 0$ for $K^i_{jlk}\, z^{hm} = 0$ implies $K^i_{jlk} = 0$.

Thus, we conclude

Theorem 4.1. *In K^h-BR-affinely connected space, Cartan's fourth curvature tensor $K^{\ i}_{jkh}$ is decomposable in form (4.2) provided the recurrent covariant tensor filed of second order is skew-symmetric.*

Transvecting (4.2) by g_{ir} and using (1.10), we get

(4.3) $K_{jrlk} = a_{lk}X_{jr}$, where $X_{jr} = g_{ir}\, X^i_j$.

Thus, we conclude

Theorem 4.2. *In K^h-BR-*affinely connected space, the associate tensor K_{jrlk} of Cartan's *fourth curvature tensor K^i_{jkh} is decomposable in the form (4.3*) provided that the recurrent covariant tensor filed of second order is skew-symmetric.

Transvecting the equation $a_{lk}\, H^i_{jhm} - a_{hm}\, H^i_{jlk} = 0$, {(3.16), [8]} by the non-zero contravariant tensor z^{hm} such that $a_{hm}\, z^{hm} = \phi \neq 0$ (non-zero scalar, i.e. tensor of zero order), we get

(4.4) $\quad \phi\, H^i_{jlk} = a_{lk}\, H^i_{jhm} z^{hm}$.

In view of quotient low, (4.4) can be written as

(4.5) $\quad H^i_{jlk} = a_{lk} \Upsilon^i_j$,

Where $\Upsilon^i_j = \frac{H^i_{jhm}\, z^{hm}}{\phi}$, since $H^i_{jhm}\, z^{hm} \neq 0$ for $H^i_{jhm}\, z^{hm} = 0$ implies $H^i_{jhm} = 0$.

Thus, we conclude

Theorem 4.3. *In K^h-BR-affinely connected space, Berward curvature tensor H^i_{jlk} is decomposable in the form (4.5) provides the recurrent covariant tensor filed of second order is skew-symmetric.*

Transvecting (4.5) by g_{ir} and using (1.17), we get

(4.6) $\quad H_{jrlk} = a_{lk} \Upsilon_{jr}$, where $\Upsilon_{jr} = g_{ir} \Upsilon^i_j$.

Thus, we conclude

Theorem 4.4. *In K^h-BR-affinely connected space, the associate curvature tensor H_{jrlk} of Berward curvature tensor H^i_{jlk} is decomposable in the form (4.6) provides the recurrent covariant tensor filed of second order is skew-symmetric.*

Transvecting the equation $a_{\ell k}\, H^i_{hm} - a_{hm}\, H^i_{lk} = 0$, {(3.23), [8]}

by the non-zero contravariant tensor z^{hm} such that $a_{hm}\, z^{hm} = \phi \neq 0$ (non-zero scalar, i.e. tensor of zero order), we get

(4.7) $\quad \phi\, H^i_{lk} = a_{lk}\, H^i_{hm} z^{hm}$.

In view of quotient low, (4.7) can be written as

(4.8) $\quad H^i_{lk} = a_{lk}\, X^i$,

where $X^i = \frac{H^i_{hm}\, z^{hm}}{\phi}$, since $H^i_{hm}\, z^{hm} \neq 0$ for $H^i_{hm}\, z^{hm} = 0$ implies $H^i_{hm} = 0$.

Thus, we conclude

Theorem 4.5. I*n K^h-BR-affinely connected space, the torsion tensor H^i_{lk} is decomposable in the form(4.8) provides the recurrent covariant tensor filed of second order is skew-symmetric.*

Transvecting the equation $a_{\ell k}\, R^i_{jhm} - a_{hm}\, R^i_{jlk} = 0$, {(3.30), [8]}

by the non-zero contravariant tensor z^{hm} such that $a_{hm}\, z^{hm} = \phi \neq 0$ (non-zero scalar, i.e. tensor of zero order), we get

(4.9) $\quad \phi\, R^i_{jlk} = a_{lk}\, R^i_{jhm}\, z^{hm}$.

In view of quotient low, (4.9) can be written as

(4.10) $\quad R^i_{jlk} = a_{lk} \mathrm{X}^i_j$,

where $\mathrm{X}^i_j = \frac{R^i_{jhm}\, z^{hm}}{\phi}$, since $R^i_{jhm}\, z^{hm} \neq 0$ for $R^i_{jhm}\, z^{hm} = 0$ implies $R^i_{jhm} = 0$.

Thus, we conclude

Theorem 4.6. *In K^h-BR-affinely connected space, Cartan's third curvature tensor R^i_{jkh} is decomposable in the form (4.10) provided that the recurrent covariant tensor filed of second order is skew-symmetric.*

Transvecting (4.10) by g_{ir} and using (1.14), we get

(4.11) $\quad R_{jrlk} = a_{lk} R_{jr}$, where $X_{jr} = g_{ir}\, X^i_j$.

Thus, we conclude

Theorem 4.7. *In K^h-BR-affinely connected space, the associate tensor R_{jrlk} of Cartan's third curvature tensor R_{jrlk} is decomposable in the form (4.11) provided that the recurrent covariant tensor filed of second order is skew-symmetric.*

B. B. Sinha considered the decomposition

(4.12) $H^i_{kh} = A^i \emptyset_{kh}$,

where $\emptyset_{kh}$ is non-zero homogenous tensor filed of the first degree in y^i and A^i is a non-zero vector filed independent of y^i.

Differentiating (4.12) partially with respect to y^i and using (1.16), we get

(4.13) $H^i_{jkh} = (\dot{\partial}_j A^i)\emptyset_{kh} + A^i(\dot{\partial}_j \emptyset_{kh})$.

Since A^i is independent of y^i, i.e. $\dot{\partial}_j A^i = 0$. Therefore (4.13) can be written as

(4.14) $H^i_{jkh} = A^i \emptyset_{jkh}$, where $\dot{\partial}_j \emptyset_{kh} = \emptyset_{jkh}$.

In K^h-*BR*-affinely connected space and in view of the equation

$a_{\ell m} H^i_{kh} + a_{lh} H^i_{mk} + a_{lk} H^i_{hm} = 0$, {(1.26), [7]; (3.17), [8]}

and using the decomposition (4.12), we get

(4.15) $a_{\ell m} A^i \emptyset_{kh} + a_{\ell h} A^i \emptyset_{mk} + a_{\ell k} A^i \emptyset_{hm} = 0$

which can be written as

(4.16) $a_{\ell m} \emptyset_{kh} + a_{\ell h} \emptyset_{mk} + a_{\ell k} \emptyset_{hm} = 0$.

Differentiating (4.16) partially with respect to y^j and if the covariant tensor filed a_{lr} is independent of y^j ,we get

(4.17) $a_{\ell m} \emptyset_{jkh} + a_{\ell h} \emptyset_{jmk} + a_{\ell k} \emptyset_{jhm} = 0$.

Thus, we conclude

Theorem 4.8. *In K^h-BR-affinely connected space, the identities (4.16) and (4.17) hold good under the decomposition (4.14), provided that the covariant tensor filed a_{lr} is independent of a y^j.*

By using the decomposition (4.14), Bianchi identity (1.17) for Berward curvature tensor H^i_{jlk} can be written as

(4.18) $A^i \emptyset_{jkh} + A^i \emptyset_{hjk} + A^i \emptyset_{khj} = 0$

which can be written as

(4.19) $\emptyset_{jkh} + \emptyset_{hjk} + \emptyset_{khj} = 0$.

Thus, we conclude

Theorem 4.9. *In K^h-BR-affinely connected space, Bianchi identity for Berward curvature tensor H^i_{jlk} can be written as (4.19)under the decomposition(4.14).*

By using (4.14) in the equation

$H^i_{jkh|m|\ell} = a_{\ell m} H^i_{jkh}$, { (2.8), [7] } ,

we get

(4.20) $(A^i \emptyset_{jkh})_{|m|\ell} = a_{\ell m}(A^i \emptyset_{jkh})$.

If A^i is covariant constant with respect to x^m , (4.20) can be written as

(4.21) $\emptyset_{jkh_{|m|\ell}} = a_{\ell m} \emptyset_{jkh}$,since$A^i \neq 0$.

Thus, we conclude

Theorem 4.10. *In K^h-BR-affinely connected space, the decomposable tensor filed $\emptyset_{jkh}$ behaves as h-BR provided that A^i is covariant constant.*

Transvecting (4.21) by y^j and using (1.2), we get

(4.22) $\emptyset_{kh_{|m|\ell}} = a_{\ell m} \emptyset_{kh}$,since$\emptyset_{jkh} y^j = \emptyset_{kh}$.

Thus, we conclude

Theorem 4.11. *In K^h-BR-affinely connected space, the decomposable tensor filed $\emptyset_{jkh}$ behaves as h-BR provided that A^i is covariant constant.*

Also, B.B. Sinha and G. Singh [11] considered the decomposition of Berward curvature tensor H^i_{jkh} in another form

(4.23) $H^i_{jkh} = T^i_j \Psi_{kh}$,

where Ψ_{kh} is a decomposable tensor filed and T_j^i is a non-zero tensor filed.

In K^h-*BR*-affinely connected space, in view of the equation

$$a_{\ell m} H_{jkh}^i + a_{lh} H_{jmk}^i + a_{lk} H_{jhm}^i = 0 \quad , \quad \{(2.25), [7]; (3.10), [8]\}$$

and by using the decomposition (4.23), we get

(4.24) $$a_{\ell m} T_j^i \Psi_{kh} + a_{lh} T_j^i \Psi_{mk} + a_{lk} T_j^i \Psi_{hm} = 0$$

which can be written as

(4.25) $$a_{\ell m} \Psi_{kh} + a_{lh} \Psi_{mk} + a_{lk} \Psi_{hm} = 0 \quad , \quad \text{since } T_j^i \neq 0 \ .$$

Thus, we conclude

Theorem 4.12. *In K^h-BR-affinely connected space, the identity (4.25) holds good under the decomposition (4.24).*

By using the decomposition (4.24), Bianchi identity (1.18) for Berward curvature tensor H_{jlk}^i can be written as

(4.26) $$T_j^i \Psi_{kh} + T_h^i \Psi_{jk} + T_k^i \Psi_{hj} = 0 \ .$$

Transvecting (4.26) by y_i , we get

(4.27) $$\beta_j \Psi_{kh} + \beta_h \Psi_{jk} + \beta_k \Psi_{hj} = 0 \quad , \text{where } T_j^i y_i = \beta_j \ .$$

Thus, we conclude

Theorem 4.13. *In K^h-BR-affinely connected space, Bianchi identity for Berwald curvature tensor H_{jlk}^i can be written as(4.27)under the decomposition (4.24).*

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أنواع مختلفة من التحليل لبعض المعاودات في K^h-BR-F_n والفضاء المتصل K^h-BR- affinely

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الملخص

في هذه الورقة، عرفنا فضاء فنسلر F_n الذي يكون فيه الموتر التقوسي الرابع لكارتان K^i_{jkh} يحقق في مفهوم كارتان ثنائي المعاودة العلاقة الاتية:

$$K^i_{jkh|m|\ell} = a_{\ell m} K^i_{jkh} \ , \qquad K^i_{jkh} \neq 0$$

كذلك قدمنا بعض التحليلات الخاصة للموترات التقوسية في فضاء فنسلر لكل من الموتر التقوسي لبروالد H^i_{jkh} و الموتر التقوسي الثالث R^i_{jkh} والرابع K^i_{jkh} لكارتان ثنائية المعاودة الهدف من هذه الورقة تمثل مناقشة التحليلات للمؤثراتالمختلفة لكلَ من فضاء فنسلر ثنائي المعاودة للموتر التقوسي K^i_{jkh} وفضاء فنسلرثنائي المعاودة K^h-BR- affinely connected space كذلك دراسة التحليلات المختلفة للموترات التقوسية الرابعة والثالثة لكارتان في K^h-BR-F_n و الموتر التقوسي لبروالد في K^h-BR-Affinely Connected Space وحصلنا على العديد من النتائج، الصيغ، المبرهنات والمتطابقات المختلفة لهذه التقوسات في هذه الفضاءات.

الكلمات المفتاحية: تحليل الموتر التقوسي، الفضاء المتصل K^h-BR-affinely، تحليل الموتر التقوسي الرابع لكارتان و تحليل الموتر التقوسي لبروالد.