A general class of generating functions of biorthogonal polynomials

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Abstract

In this paper we have obtained a new and known general class of bilateral and bilinear generating functions involving modified Konhauser biorthogonal $Y_s^{\alpha}(x;k)$, modified Bessel $Y_s^{(n)}(u)$ and Laguerre polynomials $L_n^{(\alpha)}(x)$ by group theoretic method. As in particular cases we have obtained bilinear and unilateral generating functions. Consequently we recover the result of Rainville, Srivastava Manocha and McBride [25, 29, 23] and notice that the result of Das and Chatterjea [13] is the particular case of our result.

Key words: Biorthogonal polynomials, Laguerre polynomials, modified Bessel polynomials, Generating functions & Group theoretic-method.

Introduction:

In a theoretical connection with the unification of generating functions has great importance in the study of special functions. With the steps forward in this directions has been made by some researchers [28, 7, 8, 9]. Also, the special functions have a great deal with applications in pure and applied mathematics. They appear in different frameworks. They are often used in combinatorial analysis [26], and even in statistics [20]. Moreover, the Laguerre polynomials have been applied in many other contexts, such as the Blissard problem (see [28]), the representation of Lucas polynomials of the first and second kinds [4, 17], the representation formulas of Newton sum rules for polynomials zeros [18, 19], the recurrence relations for a class of Freud-type polynomials [3], the representation of symmetric functions of a countable set of numbers, generalizing the classical algebraic Newton-Girard formulas [21]. Consequently they have been also used [6] in order to find reduction formulas for the orthogonal invariants of a strictly positive compact operator, deriving in a simple way so called Robert formulas [27]. In their study, Darus and Ibrahim [12] used deformed calculus to define generalized Laguerre polynomials and other special functions. Moreover, they gave the explicit representation formulas for the deformed Laguerre-type derivative of a composite function and illustration with applications,while Mukherjee [24] extend the bilateral generating function involving Jacobi polynomials derived by Chongdar [11], is well presented by group-theoretic method. Also, he had proved the existences of quasi bilinear generating function that implies the existence of a more general generating function. In their paper Alam and Chongdar [1], obtained some results on bilateral and trilateral generating functions of modified Laguerre polynomials. Furthermore, they made some comments on the results of Laguerre polynomials obtained by Das and Chatterjea [13]. Further, Banerji and Mohsen [2] established a result on generating relation involving modified Bessel polynomials. In their study [14], Desale and Qashash have introduced the bilateral generating function for the generalized modified Laguerre and Jacobi polynomials with the help of two linear partial differential operators. Further continuing their study they used the group theoretic method to obtain proper and improper partial bilateral as well as trilateral generating functions in [15, 16].

In this paper we have obtained new general class of generating functions for the generalized modified Konhauser biorthogonal $\mathrm{Y}_{n+m}^{\alpha+n}(x;k)$. Also, we have introduced the bilateral and bilinear generating functions for the generalized modified Laguerre and Bessel polynomials, which has been established by two linear partial differential operators. Consequently, we recovered the results of Rainville, Srivastava-Manocha and McBride [25, 29, 23]. Furthermore, we notice that result of Das and Chatterjea [13] is the particular case of our result.

For this aim, the following definition are required in this paper:

The Konhauser biorthogonal polynomials are defined by [5]

$$\mathrm{Y}_n^{\alpha}(x;k)=\frac{1}{n!}\sum_{i=0}^{n}\frac{x^i}{i!}\sum_{j=0}^{i}(1)^j\binom{i}{j}\left(\frac{j+\alpha+1}{k}\right)_n , \tag{1.1}$$

where $(\alpha)_n$ is the pochhammer symbol [29], $\alpha>-1$ and k is a positive integer.

In particular, we note [29, p.432]

$$\mathrm{Y}_n^{\alpha}(x;1)=L_n^{(\alpha)}(x), \tag{1.2}$$

where $L_n^{(\alpha)}(x)$ denotes the modified Laguerre polynomials defined by Rainville[25]

$$L_n^{(\alpha)}(x)=\frac{(1+\alpha)_n}{n}{}_1F_1(-n;1+\alpha;x), \tag{1.3}$$

which for $\alpha=0$ reduces to the classical Laguerre polynomials $L_n(x)$ [25].

The modified Bessel polynomials $Y_s^n(u)$ are defined by Chongdar[10]

$$Y_s^{(n)}(x)=y_s(x;n-s,\beta)=\sum_{k=0}^{s}k!\binom{s}{k}\binom{n+k-2}{k}\left(\frac{x}{\beta}\right)^k. \tag{1.4}$$

where $y_s(x;n,\beta)$ denotes the Bessel polynomials defined by Srivastava & Monacha [29, p.75].

This paper is organized in four sections. In the first section, we gave the introduction to the problem, while in section two, we develop the new general class of modified Konhauser biorthogonal, Laguerre and Bessel polynomials. Also, there we have introduced bilateral generating functions. In the third section of this article, we gave the applications to our results, and we conclude the results in section four.

Main Results

In this section, is developed the new general class of generating functions for modified Konhauser biorthogonal polynomials. Also, we introduce the bilateral generating function for modified Konhauser bi-orthogonal and modified Bessel polynomials are introduced.

Das and Chatterjea [13], have claimed that the operator R_1, obtained by double interpretations to both the index (n) and the parameter (α) of the Laguerre polynomial in Weisner's group-theoretic method. Also Majumdar [22], has studied the quasi bilinear generating function for the Laguerre polynomials. Here, we introduce the bilateral generating function for the generalized modified Konhauser biorthogonal, Laguerre and Bessel polynomials are introduced in the form of the following theorems:

Theorem 2.1 If there exists a generating function of the form

$$G(x,u,w)=\sum_{n=0}^{\infty}a_n w^n \mathrm{Y}_{n+m}^{\alpha+n}(x;k)Y_s^{(n)}(u) \tag{2.1}$$

Then

$$\exp\left(x-\frac{x}{(1-kw)^{\frac{1}{k}}}+w\beta\right)(1-kw)^{-\left(\frac{km+kn+\alpha+n+1}{k}\right)}(1-wu)^s$$

$$\times G\left(\frac{x}{(1-kw)^{\frac{1}{k}}},\frac{u}{1-uw},wz\right)=\sum_{n,p,q=0}^{\infty}a_n w^{n+p+q}\frac{(1+n+m)_p}{p!q!}\beta^q k^p z^n$$

$$\times \mathrm{Y}_{n+m+p}^{\alpha+n}(x;k)Y_s^{(n-q)}(u) \tag{2.2}$$

Proof: Let us carry forward with the following linear partial di_erential operators, which has been referred from Carliz (5), Changdar [10].

$$R_1 = xy^{-1}z\frac{\partial}{\partial x} + z\frac{\partial}{\partial y} + (k+1)y^{-1}z^2\frac{\partial}{\partial z} + y^{-1}z(1-z+km) \tag{2.3}$$

and

$$R_2 = u^2v^{-1}\frac{\partial}{\partial u} + uv^{-1}h\frac{\partial}{\partial h} + v^{-1}\beta\,. \tag{2.4}$$

So that

$$R_1\left[\mathrm{Y}_{n+m}^{\alpha+n}(x;k)y^{\alpha}z^{n}\right] = k(1+n+m)\mathrm{Y}_{n+m+1}^{\alpha+n}(x;k)y^{\alpha-1}z^{n+1} \tag{2.5}$$

and

$$R_2\left[Y_s^{(n)}(u)h^sv^n\right] = \beta Y_s^{(n-1)}(u)h^sv^{n-1}\,. \tag{2.6}$$

Also, we have [5, 10]

$$\exp(wR_1)f(x,y,z) = (1-kwy^{-1}z)^{-(\frac{km+1}{k})}\exp(x - \frac{x}{(1-kwy^{-1}z)^{\frac{1}{k}}}$$

$$\times f\left(\frac{x}{(1-kwy^{-1}z)^{\frac{1}{k}}}, \frac{y}{(1-kwy^{-1}z)^{\frac{1}{k}}}, \frac{z}{(1-kwy^{-1}z)^{\frac{1}{k}}}\right) \tag{2.7}$$

and

$$\exp(wR_2)f(u,v,h) = \exp\left(\frac{w\beta}{v}\right)f\left(\frac{uv}{v-wu}, v, \frac{h}{v}(v-wu)\right). \tag{2.8}$$

Now, we consider the following generating relation

$$G(x,u,w) = \sum_{n=0}^{\infty} a_n w^n \mathrm{Y}_{n+m}^{\alpha+n}(x;k)Y_s^{(n)}(u) \tag{2.9}$$

replacing w by wzv and then multiplying both sides by $y^{\alpha}h^{s}$, we get

$$y^{\alpha}h^{s}G(x,u,wzv) = y^{\alpha}h^{s}\sum_{n=0}^{\infty} a_n (wzv)^n \mathrm{Y}_{n+m}^{\alpha+n}(x;k)Y_s^{(n)}(u) \tag{2.10}$$

Operating exp(wR1) exp(wR2) on both sides of (2.10), we have

$$\exp(wR_1)\exp(wR_2)\left[y^{\alpha}h^{s}G(x,u,wzv)\right]$$

$$= \exp(wR_1)\exp(wR_2)\sum_{n=0}^{\infty} a_n w^n [\mathrm{Y}_{n+m}^{\alpha+n}(x;k)y^{\alpha}z^{n}]$$

$$\times[Y_s^{(n)}(u)h^sv^n]) \tag{2.11}$$

With the help of (2.7) and (2.8) the left hand side of (2.11) can be simplified as

$$\exp\left(x - \frac{x}{\left(1-kwy^{-1}z\right)^{\frac{1}{k}}}\right)\left(1-kwy^{-1}z\right)^{-\left(\frac{km+kn+\alpha+n+1}{k}\right)}\exp\left(\frac{w\beta}{v}\right)y^{\alpha}\left(v-wu\right)^{s}$$

$$\times G\left(\frac{x}{(1-kwy^{-1}z)^{\frac{1}{k}}}, \frac{uv}{v-uw}, wvz\right). \tag{2.12}$$

Also, the right hand side of (2.11) with the help of (2.5) and (2.6) is simplified as

$$= \sum_{n,p,q=0}^{\infty} a_n (kwy^{-1}z)^p (\beta v^{-1}w)^q (wzv)^n \frac{(1+n+m)_p}{p!q!} y^{\alpha} h^s$$

$$\times \Upsilon_{n+m+p}^{\alpha+n}(x;k) Y_s^{(n-q)}(u) \tag{2.13}$$

Therefore, the simplified form of (2.11) is

$$\exp\left(x - \frac{x}{(1-kwy^{-1}z)^{\frac{1}{k}}} + \frac{w\beta}{v}\right)(1-kwy^{-1}z)^{-\left(\frac{km+kn+\alpha+n+1}{k}\right)}(1-wuv^{-1})^s$$

$$\times G\left(\frac{x}{(1-kwy^{-1}z)^{\frac{1}{k}}}, \frac{uv}{v-uw}, wvz\right) = \sum_{n,p,q=0}^{\infty} a_n (kwy^{-1}z)^p (\beta v^{-1}w)^q (wzv)^n$$

$$\times \frac{(1+n+m)_p}{p!q!} \Upsilon_{n+m+p}^{\alpha+n}(x;k) Y_s^{(n-q)}(u) \tag{2.14}$$

Finally substituting $\frac{z}{y} = 1$ and $v = 1$ in (2.14), we obtain bilateral generating function (2.15) for generalized modified Konhauser bi-orthogonal and Bessel polynomials.

$$\exp\left(x - \frac{x}{(1-kw)^{\frac{1}{k}}} + w\beta\right)(1-kw)^{-\left(\frac{km+kn+\alpha+n+1}{k}\right)}(1-wu)^s$$

$$\times G\left(\frac{x}{(1-kw)^{\frac{1}{k}}}, \frac{u}{1-uw}, wz\right) = \sum_{n,p,q=0}^{\infty} a_n (wz)^n (w\beta)^q (wk)^p \frac{(1+n+m)_p}{p!q!}$$

$$\times \Upsilon_{n+m+p}^{\alpha+n}(x;k) Y_s^{(n-q)}(u) \tag{2.15}$$

This completes the proof of Theorem 2.1.

Theorem 2.2 If there exists bilateral generating relation of the form

$$G(x,u,w) = \sum_{n=0}^{\infty} a_n w^n \Upsilon_{n+m}^{\alpha+n}(x;k) L_n^{(\beta-n)}(u) \tag{2.16}$$

Then

$$\exp\left(x - \frac{x}{(1-kw)^{\frac{1}{k}}} - wu\right)(1-kw)^{-\left(\frac{km+kn+\alpha+n+1}{k}\right)}(1+w)^{\beta-n}$$

$$\times G\left(\frac{x}{(1-kw)^{\frac{1}{k}}}, u(1+w), w\right) = \sum_{n,p,q=0}^{\infty} a_n w^{n+p+q} k^p \frac{(1+n+m)_p (1+n)_q}{p!q!}$$

$$\times \Upsilon_{n+m+p}^{\alpha+n}(x;k) L_{n+q}^{(\beta-n-q)}(u) \tag{2.17}$$

Proof: Let us define the operator R_3

$$R_3 = uv\frac{\partial}{\partial u} - u^2\frac{\partial}{\partial v} - uv + \beta v \tag{2.18}$$

(One may concern [1] for more details about the operator R_3.) Operating R_3 on $v^n L_n^{(\beta-n)}(u)$, we get

$$R_3\left[L_n^{(\beta-n)}(u)v^n\right] = (1+n)L_{n+1}^{(\beta-n+1)}(u)v^{n+1} \tag{2.19}$$

Also, we have [1]

$$\exp(wR_3)f(u,v) = \exp(-wuv)(1+wv)^{\beta} f\left(u(1+wv), \frac{v}{1+wv}\right). \tag{2.20}$$

Now we consider the generating relation

$$G(x,u,w) = \sum_{n=0}^{\infty} a_n w^n \mathrm{Y}_{n+m}^{\alpha+n}(x;k)L_n^{(\beta-n)}(u) \tag{2.21}$$

In above relation replacing w by wzv and then multiplying both sides by y^{α}, we get

$$y^{\alpha}G(x,u,wvz) = y^{\alpha}\sum_{n=0}^{\infty} a_n (wvz)^n \mathrm{Y}_{n+m}^{\alpha+n}(x;k)L_n^{(\beta-n)}(u) \tag{2.22}$$

Operating exp(wR1) exp(wR3) on both sides of (2.22), we have

$$\exp(wR_1)\exp(wR_3)\left[y^{\alpha}G(x,u,wzv)\right]$$
$$= \exp(wR_1)\exp(wR_3)\sum_{n=0}^{\infty} a_n (wvz)^n \mathrm{Y}_{n+m}^{\alpha+n}(x;k)y^{\alpha}L_n^{(\beta-n)}(u) \tag{2.23}$$

With the help of (2.5) and (2.19) the right hand side of (2.23) can be simplified as

$$\sum_{n,p,q=0}^{\infty} a_n w^{n+p+q} k^p \frac{(1+n+m)_p (1+n)_q}{p!q!} \mathrm{Y}_{n+m+p}^{\alpha+n}(x;k)L_{n+q}^{(\beta-n-q)}(u)y^{\alpha-p}z^{n+p}v^{n+q} \tag{2.24}$$

Also, the left hand side of (2.23) with the help of (2.20) and (2.7) is simplified as

$$\exp\left(x - \frac{x}{(1-kwy^{-1}z)^{\frac{1}{k}}}\right)(1-kwy^{-1}z)^{-\left(\frac{km+kn+\alpha+n+1}{k}\right)} \exp(-wuv)y^{\alpha}z^n(1+wv)^{\beta}$$
$$\times G\left(\frac{x}{(1-kwy^{-1}z)^{\frac{1}{k}}}, u(1+wv), \frac{vzw}{1+wv}\right). \tag{2.25}$$

Therefore, the simplified form of (2.23) is

$$\exp\left(x - \frac{x}{(1-kwy^{-1}z)^{\frac{1}{k}}}\right)(1-kwy^{-1}z)^{-\left(\frac{km+kn+\alpha+n+1}{k}\right)} \exp(-wuv)(1+wv)^{\beta}$$
$$\times G\left(\frac{x}{(1-kwy^{-1}z)^{\frac{1}{k}}}, u(1+wv), \frac{vzw}{1+wv}\right) = \sum_{n,p,q=0}^{\infty} a_n w^{n+p+q}(ky^{-1}z)^p v^{n+q}$$
$$\times \frac{(1+n+m)_p(1+n)_q}{p!q!}\mathrm{Y}_{n+m+p}^{\alpha+n}(x;k)L_{n+q}^{(\beta-n-q)}(u) \tag{2.26}$$

Finally substituting $z = y = v = 1$ in (2.26), we arrive at the proof of Theorem 2.2.

Application

I. In Theorem 2.1, if we put $k=1$ and use (1.2), we obtain the following result which we hope a new bilateral generating function for Laguerre polynomials.

Corollary 3.1 If there exists a bilateral generating function

$$G(x,u,w)=\sum_{n=0}^{\infty} a_n w^n L_{n+m}^{(\alpha+n)}(x) Y_s^{(n)}(u)\ , \tag{3.1}$$

then the following more general bilateral generating relation holds

$$\exp\left(x-\frac{x}{(1-w)}+w\beta\right)(1-w)^{-(m+2n+\alpha+1)}(1-wu)^s G\left(\frac{x}{(1-w)},\frac{u}{1-uw},wz\right)$$
$$=\sum_{n,p,q=0}^{\infty} a_n w^{n+p+q}\frac{(1+n+m)_p}{p!q!}\beta^q z^n L_{n+m+p}^{(\alpha+n)}(x) Y_s^{(n-q)}(u). \tag{3.2}$$

As an application of Corollary 3.1, if we put $s=m=0$ in (3.2) and take in consideration that $G(x,u,w)$ becomes $G(x,w)$ for $Y_0^{(n-q)}(u)=1$, we obtain the following generating relation:

$$(1-w)^{-(2n+\alpha+1)}\exp\left(\frac{-wx}{1-w}\right)G\left(\frac{x}{1-w},wz\right)=\sum_{n,p=0}^{\infty} a_n w^{n+p}\frac{(1+n)_p}{p!} z^n L_{n+p}^{(\alpha+n)}(x) \tag{3.3}$$

Next, if we put $a_n=1$ and replacing α by $\alpha-n$ in (3.3) , we get

$$(1-w)^{-(n+\alpha+1)}\exp\left(\frac{-wx}{1-w}\right)L_n^{(\alpha)}\left(\frac{x}{1-w}\right)=\sum_{n=0}^{\infty}\sum_{p=0}^{\infty} w^p\frac{(n+p)!}{n!p!}L_{n+p}^{(\alpha)}(x) \tag{3.4}$$

which is obtained by [25, 29, 23].

Further, if we put $n=0$ in (3.4) we get

$$(1-w)^{-(\alpha+1)}\exp\left(\frac{-wx}{1-w}\right)=\sum_{p=0}^{\infty} w^p L_p^{(\alpha)}(x) \tag{3.5}$$

which is obtained by Mckherjee[25], Raiville[23].

Again, if we put $\alpha=0$, in (3.4), we obtain the following generating function for the classical Laguerre polynomial Mckherjee [25]

$$(1-w)^{-(n+1)}\exp\left(\frac{-wx}{1-w}\right)L_n\left(\frac{x}{1-w}\right)=\sum_{n=0}^{\infty}\sum_{p=0}^{\infty} w^p\frac{(n+p)!}{n!p!}L_{n+p}(x). \tag{3.6}$$

II. In Theorem 2.2, if we put $k=1$ and $m=0$ and use (1.2) we obtain the following result which we hope a new bilinear generating function for Laguerre polynomials.

Corollary 3.2 If there exists a bilinear generating function

$$G(x,u,w)=\sum_{n=0}^{\infty} a_n w^n L_n^{(\alpha+n)}(x) L_n^{(\beta-n)}(u), \tag{3.7}$$

then the following generating relation holds.

$$\exp\left(\frac{-wx}{(1-w)}-wu\right)(1-w)^{-(2n+\alpha+1)}(1+wu)^{\beta-n}G\left(\frac{x}{1-w},u(1+w),w\right)$$

$$=\sum_{n,p,q=0}^{\infty}a_n w^{n+p+q}\frac{(1+n)_p}{p!}\frac{(1+n)_q}{q!}L_{n+p}^{(\alpha+n)}(x)L_{n+q}^{(\beta-n-q)}(u). \tag{3.8}$$

Now, if we replace α by $\alpha-n$ and β by $\beta+n$ in (3.8), we get

$$\exp\left(\frac{-wx}{(1-w)}-wu\right)(1-w)^{-(n+\alpha+1)}(1+w)^{\beta}G\left(\frac{x}{1-w},u(1+w),w\right)$$

$$=\sum_{n,p,q=0}^{\infty}a_n w^{n+p+q}\binom{p+n}{p}\binom{q+n}{q}L_{n+p}^{(\alpha)}(x)L_{n+q}^{(\beta-q)}(u). \tag{3.9}$$

Also, if we put $a_n=1$ in (3.9) and consider the following generating relation Mckherjee [25]:

$$\exp\left(\frac{-wx}{1-w}\right)(1-w)^{-(n+\alpha+1)}L_n^{\alpha}\left(\frac{x}{1-w}\right)=\sum_{p=0}^{\infty}w^p\binom{p+n}{p}L_{n+p}^{(\alpha)}(x) \tag{3.10}$$

then we get

$$\exp(-wu)(1+w)^{\beta}L_n^{\beta}(u(1+w))=\sum_{q=0}^{\infty}w^q\binom{q+n}{q}L_{n+q}^{(\beta-q)}(u) \tag{3.11}$$

which is as same as obtained by Das and Chatterjea in their paper [13].

Again, if we put $a_n=1$ and $n=0$ in (3.9), we get

$$\exp\left(\frac{-wx}{(1-w)}-wu\right)(1-w)^{-(\alpha+1)}(1+w)^{\beta}=\sum_{p,q=0}^{\infty}w^{p+q}L_p^{(\alpha)}(x)L_q^{(\beta-q)}(u). \tag{3.12}$$

Which we hope a new bilinear generating function for Laguerre polynomials.

Conclusion

In this paper, we have introduced a new general class of generating functions in the form of (2.2) and (2.17) for Konhauser biorthogonal, Bessel polynomials and Konhauser biorthogonal, Laguerre polynomials respectively. In section 3, we have obtained some particular cases of generating functions (2.2) and (2.17) for k = 1, which is a new class of bilateral and bilinear generating functions (3.2) and (3.8) that involves Laguerre, Bessel polynomials and Laguerre polynomials respectively. Also, the particular case of Corollary 3.1 for $s=m=0$ is an unilateral generating function (3.3) and thereby substituting $a_n=1$ and replacing α by $\alpha-n$, we recover the result obtained by [25, 29, 23]. Also, we have shown that Das and Chaterjea's [13] (one may refer) result is a particular case of Corollary 3.3 for $a_n=1$ and consider generating relation (3.10).

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فئه عامة من الدوال المولدة لكثيرات الحدود ثنائية التعامد

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الملخص

في هذه الورقة حصلنا على فئة عامة جديدة ومعرفة لدوال مولدة ثنائية ومترابطة التي تحتوي كثيرات الحدود كونهوسر المعدلة، بسل المعدلة ولاجير باستعمال طريقة الزمر النظري. وكحالات خاصة حصلنا على دوال مولدة مترابطة وأحادية، ومنها أستعدنا النتيجة التي حصل عليها رينفيلا, سرفستافا- مانوشا، وماسبريد (راجع 25, 29, 23), ونلاحظ النتيجة التي حصل عليها داس- وتشاترجي هي حالة خاصة من نتيجتنا (راجع 13).

الكلمات المفتاحية: كثيرات الحدود ثنائية التعامد، كثيرات حدود لاجير، كثيرات حدود بسل المعدلة، الدوال المولدة، طريقة الزمر النظري.