Analytic solutions for a new model of the(3+1)-Boussinesq equation

Mubarak A.H. Alqufail, M.S. Al-Amry and Buthaina M.M. S.Own
Department of Mathematics, Faculty of Education, Aden University, Aden, Yemen
DOI: https://doi.org/10.47372/uajnas.2019.n2.a17

Abstract

In this paper, we have applied the mapping method to solve the (3+1)-dimensional Boussinenesq equation where we have obtained exact solutions for evolution equation to construct exact periodic and soliton solutions of nonlinear partial differential evolution equation. Many have obtained new families of exact traveling wave solutions, but the Boussinesq equation is successfully. These solutions may be significantly important for the explanation of some practical physical problems.

New exact travelling wave solutions are obtained and expressed in terms of hyperbolic functions, trigonometric functions, rational functions and elliptic functions. It is shown that the mapping method provides a powerful mathematical tool for solving a great many nonlinear partial differential equations in mathematical physics.

Key words: Mapping Method, Exact Solutions, The (3+1)-Boussinenesq Equation.

Introduction

Seeking exact solutions of nonlinear evolution equations (NLEEs) is very important in mathematical physics becomes it is one of the most exciting and extremely active areas of research investigation. In the past several decades, manyeffective methods for obtaining exact solutions of (NLEEs) have been presented [1], such as the tanh method [13,15,18], the extended tanh method [19,20,21], the sine–cosine method [16,17], the homogeneous balance method [6,7], the exp-function method [2,3,9,10,11] and the Hirotamethod [22,23] which has been used to investigate nonlinear dispersive and dissipative problems.

Consider the (3 + 1)-dimensional Boussinesq equation

$$u_{tt} - u_{xx} - u_{yy} - u_{zz} - (u^2)_{xx} - u_{xxxx} = 0$$

More new double periodic and multiple soliton solutions are obtained for the generalized (3 + 1)-dimensional Boussinesq equation in [4]. Chen et al. [5] studied (3 + 1) -dimensional Boussinesq equation by using the new generalized transformation in homogeneous balance method. Feng et al. [8] have investigated the bifurcations and global dynamic behavior of two variants of (3 + 1)-dimensional Boussinesq-type equations with positive and negative exponents and obtained the sufficient conditions under which solitary, kink, breaking and periodic wave solutions appear. A mathematical method is constructed to study two variants of the two-dimensional Boussinesq water equation with positive and negative exponents in [12].

Description of the Mapping Method

Consider the general nonlinear partial differential equations (NLPDEs), say, in two variables,

$$P\left(u, u_x, u_t, u_{xx}, u_{yy}, u_{zz}, u_{xt}, \dots\right) = 0, \qquad (1)$$

where $u = u(x, y, z, t)$ is an unknown function. P is a polynomial in $u(x, y, z, t)$.

Step 1.

Let $u(x, y, z, t) = u(\xi), \xi = \lambda(x + y + z - ct)$ where λ and c are constant,then equation (1) reduces to a nonlinear ordinary differential equation (NLODE)

$$Q(u, u', u'', \dots) = 0, \qquad (2)$$

where the superscripts stand for the ordinary derivatives with respect to (ξ).

Step 2.

Assume the solution of equation (2) takes the form

$$u(x,t) = u(\xi) = a_0 + \sum_{i=1}^{m} a_i\big(f(\xi)\big)^i + b_i\big(f(\xi)\big)^{-i}, \tag{3}$$

where the coefficients a_0, a_i, b_i and $f = f(\xi)$ satisfies a nonlinear ordinary differential equation

$$\frac{df(\xi)}{d\xi} = \sqrt{pf^2(\xi) + \frac{1}{2}qf^4(\xi) + r} \quad , \quad p, q, r \in R. \tag{4}$$

Step 3. The parameter $m$will be found by balancingthe highest-order nonlinear term with the highest-order partial derivative termin the given equation.

Step4.

Substituting (3) into (2), using (4) repeatedly and setting the coefficients of each order of $f^i(\xi), f^i(\xi)\sqrt{pf^2(\xi) + \frac{1}{2}qf^4(\xi) + r}$ to zero, we obtain a set of nonlinear algebraic equations for a_0, a_i, b_i, λ and c.

Step5.

With the aid of the computer program Maple, we can solve the set of nonlinear algebraic equations and obtain all the constantsa_0, a_i, b_i, λ and c.

Step6.

The (NLODE) (4) has the following solutions

1. $f(\xi) = sech(\xi), [p = 1, q = -2, r = 0]$,
2. $f(\xi) = tanh(\xi), [p = -2, q = 2, r = 1]$,
3. $f(\xi) = \frac{1}{2}\ tanh(2\xi) or\ \frac{1}{2}\ coth(2\xi), [p = -8, q = 32, r = 1]$,
4. $f(\xi) = \frac{1}{2}\ tan(2\xi) or\ \frac{1}{2}\ cot(2\xi), [p = 8, q = 32, r = 1]$,
5. $f(\xi) = sn\xi, [p = -(k^2+1), q = 2k^2, r = 1]$,
6. $f(\xi) = ns\xi, [p = -(k^2+1), q = 2, r = k^2]$,
7. $f(\xi) = cd\xi, [p = -(k^2+1), q = 2k^2, r = 1]$,
8. $f(\xi) = dc\xi, [p = -(k^2+1), q = 2, r = k^2]$,
9. $f(\xi) = cn\xi, [p = 2k^2 - 1, q = -2k^2, r = 1 - k^2]$,
10. $f(\xi) = nc\xi, [p = 2k^2 - 1, q = 2(1-k^2), r = -k^2]$,
11. $f(\xi) = dn\xi, [p = 2 - k^2, q = -2, r = -(1-k^2)]$,
12. $f(\xi) = nd\xi, [p = 2 - k^2, q = 2(k^2-1), r = -1]$,
13. $f(\xi) = cs\xi, [p = 2 - k^2, q = 2, r = 1 - k^2]$,
14. $f(\xi) = sc\xi, [p = 2 - k^2, q = 2(1-k^2), r = 1]$,
15. $f(\xi) = ds\xi, [p = -1 + 2k^2, q = 2, r = -k^2(1-k^2)]$,
16. $f(\xi) = sd\xi, [p = -1 + 2k^2, q = 2k^2(k^2-1), r = 1]$,

The multiple exact special solutions of nonlinear partial differential equation (1) are obtained by making use of (3) and the solutions of (NLODE)(4).

Where $sn\,\xi = sn\,(\xi,k), cn\,\xi = cn\,(\xi,k)$ and $dn\,\xi = dn\,(\xi,k)$ are the Jacobi elliptic function and k $(0 \le k^2 \le 1)$ is the modulus of these functions.

Application

In this section, we present our proposed (3+1)-Boussinenesq equation as the form

$$u_{tt} - u_{xx} - u_{yy} - u_{zz} - (u^2)_{xx} - u_{xxxx} = 0, \tag{5}$$

The(3+1)-Boussinseq equation[14] describes motions of long waves in shallow water under gravity and in a one-dimensional nonlinear lattice.

Now, we apply the mapping method to solve the (3+1)-Boussinseq equation. Consequently, we get the original solutions for the (3+1)-Boussinseq equation, as follows:

Substituting $u(x,y,z,t) = u(\xi), \xi = \lambda\,(x + y + z - ct)$ into (5) gives

$(c^2-3)u''-(u^2)''-\lambda^2 u^{(4)}=0,$ (6)
where integrating twice yields
$(c^2-3)u-u^2-\lambda^2 u''=0,$ (7)

Balancing the order of the nonlinear term u^2 with the highest derivative term u'' gives $2m=m+2$ that gives $m=2$.
Assume the solution of (7) has the form

$$u(\xi)=a_0+a_1 f(\xi)+a_2 f(\xi)^2+b_1 f(\xi)^{-1}+b_2 f(\xi)^{-2}, \quad (8)$$

where $\frac{df(\xi)}{d\xi}=\sqrt{pf^2(\xi)+\frac{1}{2}qf^4(\xi)+r} \quad , \quad p,q,r\in R.$ (9)

Substituting (8) in (7) and using (9), collecting the coefficients of each power of $f^i, 0\le i\le 8$. Setting each coefficient to zero and solving the resulting system with the aid of the computer program Maple, we obtain the following sets of solutions.

1. $a_0=a_0, a_1=0, a_2=0, b_1=0, b_2=0, c=c, \lambda=\lambda,$
2. $a_0=-2\lambda^2 p+2\sqrt{\lambda^4p^2+6\lambda^4qr}, a_1=0, a_2=-3\lambda^2 q, b_1=0,$
$b_2=-6\lambda^2 r, \quad c=\pm\sqrt{3+4\sqrt{\lambda^4p^2+6\lambda^4qr}}, \quad \lambda=\lambda,$
3. $a_0=-2\lambda^2 p-2\sqrt{\lambda^4p^2+6\lambda^4qr}, a_1=0, a_2=-3\lambda^2 q, b_1=0,$
$b_2=-6\lambda^2 r, \quad c=\pm\sqrt{3-4\sqrt{\lambda^4p^2+6\lambda^4qr}}, \quad \lambda=\lambda,$
4. $a_0=-2\lambda^2 p+\sqrt{4\lambda^4p^2-6\lambda^4qr}, a_1=0, a_2=0, b_1=0, b_2=-6\lambda^2 r,$
$c=\pm\sqrt{3+2\sqrt{4\lambda^4p^2-6\lambda^4qr}}, \lambda=\lambda,$
5. $a_0=-2\lambda^2 p-\sqrt{4\lambda^4p^2-6\lambda^4qr}, a_1=0, a_2=0, b_1=0, b_2=-6\lambda^2 r,$
$c=\pm\sqrt{3-2\sqrt{4\lambda^4p^2-6\lambda^4qr}}, \lambda=\lambda,$
6. $a_0=-2\lambda^2 p+\sqrt{4\lambda^4p^2-6\lambda^4qr}, a_1=0, a_2=-3\lambda^2 q, b_1=0, b_2=0,$
$c=\pm\sqrt{3+2\sqrt{4\lambda^4p^2-6\lambda^4qr}}, \lambda=\lambda,$
7. $a_0=-2\lambda^2 p-\sqrt{4\lambda^4p^2-6\lambda^4qr}, a_1=0, a_2=-3\lambda^2 q, b_1=0, b_2=0,$
$c=\pm\sqrt{3-2\sqrt{4\lambda^4p^2-6\lambda^4qr}}, \lambda=\lambda.$

Using (8), the solution of (9) when $[p=1, q=-2, r=0]$ and the sets of solutions (1)-(7), we get

$$u_1(x,y,z,t)=a_0, \quad a_0\in R$$

$$u_{2,3}(x,y,z,t)=6\lambda^2\left(\operatorname{sech}\left(\lambda\left(x+y+z\pm\sqrt{3+4\lambda^2}\,t\right)\right)\right)^2,$$

$$u_{4,5}(x,y,z,t)=-4\lambda^2+6\lambda^2\left(\operatorname{sech}\left(\lambda\left(x+y+z\pm\sqrt{3-4\lambda^2}\,t\right)\right)\right)^2.$$

Using (8), the solution of (9) when $[p=-2, q=2, r=1]$ and the sets of solutions (2)-(7), we get

$$u_{6,7}(x,y,z,t)=12\lambda^2-6\lambda^2\left(\tanh\left(\lambda\left(x+y+z\pm\sqrt{3+16\lambda^2}\,t\right)\right)\right)^2 -6\lambda^2\left(\coth\left(\lambda\left(x+y+z\pm\sqrt{3+16\lambda^2}\,t\right)\right)\right)^2,$$

$$u_{8,9}(x,y,z,t)=-4\lambda^2-6\lambda^2\left(\tanh\left(\lambda\left(x+y+z\pm\sqrt{3-16\lambda^2}\,t\right)\right)\right)^2 -6\lambda^2\left(\coth\left(\lambda\left(x+y+z\pm\sqrt{3-16\lambda^2}\,t\right)\right)\right)^2,$$

$$u_{10,11}(x,y,z,t) = 6\lambda^2 - 6\lambda^2\left(\coth\left(\lambda\left(x+y+z \pm \sqrt{3+4\lambda^2}\,t\right)\right)\right)^2,$$
$$u_{12,13}(x,y,z,t) = 2\lambda^2 - 6\lambda^2\left(\coth\left(\lambda\left(x+y+z \pm \sqrt{3-4\lambda^2}\,t\right)\right)\right)^2,$$
$$u_{14,15}(x,y,z,t) = 6\lambda^2 - 6\lambda^2\left(\tanh\left(\lambda\left(x+y+z \pm \sqrt{3+4\lambda^2}\,t\right)\right)\right)^2,$$
$$u_{16,17}(x,y,z,t) = 2\lambda^2 - 6\lambda^2\left(\tanh\left(\lambda\left(x+y+z \pm \sqrt{3-4\lambda^2}\,t\right)\right)\right)^2.$$

Using (8), the solution of (9) when $[p=-8, q=32, r=1]$ and the sets of solutions (2)-(7), we get

$$u_{18,19}(x,y,z,t) = 48\lambda^2 - 24\lambda^2\left(\tanh\left(2\lambda\left(x+y+z \pm \sqrt{3+64\lambda^2}\,t\right)\right)\right)^2 - 24\lambda^2\left(\coth\left(2\lambda\left(x+y+z \pm \sqrt{3+64\lambda^2}\,t\right)\right)\right)^2,$$
$$u_{20,21}(x,y,z,t) = -16\lambda^2 - 24\lambda^2\left(\tanh\left(2\lambda\left(x+y+z \pm \sqrt{3-64\lambda^2}\,t\right)\right)\right)^2 - 24\lambda^2\left(\coth\left(2\lambda\left(x+y+z \pm \sqrt{3-64\lambda^2}\,t\right)\right)\right)^2,$$
$$u_{22,23}(x,y,z,t) = 24\lambda^2 - 24\lambda^2\left(\coth\left(2\lambda\left(x+y+z \pm \sqrt{3+16\lambda^2}\,t\right)\right)\right)^2,$$
$$u_{24,25}(x,y,z,t) = 8\lambda^2 - 24\lambda^2\left(\coth\left(2\lambda\left(x+y+z \pm \sqrt{3-16\lambda^2}\,t\right)\right)\right)^2,$$
$$u_{26,27}(x,y,z,t) = 24\lambda^2 - 24\lambda^2\left(\tanh\left(2\lambda\left(x+y+z \pm \sqrt{3+16\lambda^2}\,t\right)\right)\right)^2,$$
$$u_{28,29}(x,y,z,t) = 8\lambda^2 - 24\lambda^2\left(\tanh\left(2\lambda(x+y+z \pm \sqrt{3-16\lambda^2}\,t)\right)\right)^2.$$

Using (8), the solution of (9) when $[p=8, q=32, r=1]$ and the sets of solutions (2)-(7), we get

$$u_{30,31}(x,y,z,t) = 16\lambda^2 - 24\lambda^2\left(\tan\left(2\lambda\left(x+y+z \pm \sqrt{3+64\lambda^2}\,t\right)\right)\right)^2 - 24\lambda^2\left(\cot\left(2\lambda\left(x+y+z \pm \sqrt{3+64\lambda^2}\,t\right)\right)\right)^2,$$
$$u_{32,33}(x,y,z,t) = -48\lambda^2 - 24\lambda^2\left(\tan\left(2\lambda\left(x+y+z \pm \sqrt{3-64\lambda^2}\,t\right)\right)\right)^2 - 24\lambda^2\left(\cot\left(2\lambda\left(x+y+z \pm \sqrt{3-64\lambda^2}\,t\right)\right)\right)^2,$$
$$u_{34,35}(x,y,z,t) = -8\lambda^2 - 24\lambda^2\left(\cot\left(2\lambda\left(x+y+z \pm \sqrt{3+16\lambda^2}\,t\right)\right)\right)^2,$$
$$u_{36,37}(x,y,z,t) = -24\lambda^2 - 24\lambda^2\left(\cot\left(2\lambda\left(x+y+z \pm \sqrt{3-16\lambda^2}\,t\right)\right)\right)^2,$$
$$u_{38,39}(x,y,z,t) = -8\lambda^2 - 24\lambda^2\left(\tan\left(2\lambda\left(x+y+z \pm \sqrt{3+16\lambda^2}\,t\right)\right)\right)^2,$$
$$u_{40,41}(x,y,z,t) = -24\lambda^2 - 24\lambda^2\left(\tan\left(2\lambda(x+y+z \pm \sqrt{3-16\lambda^2}\,t)\right)\right)^2.$$

Using (8), the solution of (9) when $[p=-(k^2+1), q=2k^2, r=1]$ and the sets of solutions (2)-(7), we get

$$u_{42,43,\dots,53}(x,y,z,t) = a_0 + a_2\left(sn\left(\lambda(x+y+z-ct)\right)\right)^2 + b_2\left(ns\left(\lambda(x+y+z-ct)\right)\right)^2,$$

where a_0, a_2 *and* b_2 are defined in the sets of solutions (2)-(7).
Note that, when $k \to 1$, we obtain $[u_{6,7}(x,y,z,t), u_{8,9}(x,y,z,t), \dots, u_{16,17}(x,y,z,t)]$,
and when $k \to 0$, we obtain

$u_{54,55}(x,y,z,t) = 4\lambda^2 - 6\lambda^2\left(\csc\left(\lambda\left(x+y+z \pm \sqrt{3+4\lambda^2}\,t\right)\right)\right)^2,$

$u_{56,57}(x,y,z,t) = -6\lambda^2\left(\csc\left(\lambda\left(x+y+z \pm \sqrt{3-4\lambda^2}\,t\right)\right)\right)^2.$

Using (8), the solution of (9) when $[p = -(k^2+1), q = 2, r = k^2]$ and the sets of solutions (2)-(7), we get

$$u_{58,59,\dots,69}(x,y,z,t) = a_0 + a_2\left(ns\left(\lambda(x+y+z-ct)\right)\right)^2 + b_2\left(sn\left(\lambda(x+y+z-ct)\right)\right)^2,$$

where a_0, a_2 *and* b_2 are defined in the sets of solutions (2)-(7).

As $k \to 1$, we obtain $[u_{6,7}(x,y,z,t), u_{8,9}(x,y,z,t), \dots, u_{16,17}(x,y,z,t)]$. Also as $k \to 0$, we obtain $[u_{54,55}(x,y,z,t)$ *and* $u_{56,57}(x,y,z,t)]$.

Using (8), the solution of (9) when $[p = -(k^2+1), q = 2k^2, r = 1]$ and the sets of solutions (2)-(7), we get

$$u_{70,71,\dots,81}(x,y,z,t) = a_0 + a_2\left(cd\left(\lambda(x+y+z-ct)\right)\right)^2 + b_2\left(dc\left(\lambda(x+y+z-ct)\right)\right)^2,$$

where a_0, a_2 *and* b_2 are defined in the sets of solutions (2)-(7).

Note that, when $k \to 1$, we obtain constant solution, and when $k \to 0$, we obtain

$u_{82,83}(x,y,z,t) = 4\lambda^2 - 6\lambda^2\left(\sec\left(\lambda\left(x+y+z \pm \sqrt{3+4\lambda^2}\,t\right)\right)\right)^2,$

$u_{84,85}(x,y,z,t) = -6\lambda^2\left(\sec\left(\lambda\left(x+y+z \pm \sqrt{3-4\lambda^2}\,t\right)\right)\right)^2.$

Using (8), the solution of (9) when $[p = -(k^2+1), q = 2, r = k^2]$ and the sets of solutions (2)-(7), we get

$$u_{86,87,\dots,97}(x,y,z,t) = a_0 + a_2\left(dc\left(\lambda(x+y+z-ct)\right)\right)^2 + b_2\left(cd\left(\lambda(x+y+z-ct)\right)\right)^2,$$

where a_0, a_2 *and* b_2 are defined in the sets of solutions (2)-(7).

As $k \to 1$, we obtain constant solution. Also as $k \to 0$, we obtain $[u_{82,83}(x,y,z,t)$ *and* $u_{84,85}(x,y,z,t)]$.

Using (8), the solution of (9) when $[p = 2k^2 - 1, q = -2k^2, r = 1 - k^2]$ and the sets of solutions (2)-(7), we get

$$u_{98,99,\dots,109}(x,y,z,t) = a_0 + a_2\left(cn\left(\lambda(x+y+z-ct)\right)\right)^2 + b_2\left(nc\left(\lambda(x+y+z-ct)\right)\right)^2,$$

where a_0, a_2 *and* b_2 are defined in the sets of solutions (2)-(7).

As $k \to 1$, we obtain $[u_{2,3}(x,y,z,t)$ *and* $u_{4,5}(x,y,z,t)]$. Also as $k \to 0$, we obtain $[u_{82,83}(x,y,z,t)$ *and* $u_{84,85}(x,y,z,t)]$.

Using (8), the solution of (9) when $[p = 2k^2 - 1, q = 2(1-k^2), r = -k^2]$ and the sets of solutions (2)-(7), we get

$$u_{110,111,\dots,121}(x,y,z,t) = a_0 + a_2\left(nc\left(\lambda(x+y+z-ct)\right)\right)^2 + b_2\left(cn\left(\lambda(x+y+z-ct)\right)\right)^2,$$

where a_0, a_2 *and* b_2 are defined in the sets of solutions (2)-(7).

As $k \to 1$, we obtain $[u_{2,3}(x,y,z,t)$ *and* $u_{4,5}(x,y,z,t)]$. Also as $k \to 0$, we obtain $[u_{82,83}(x,y,z,t)$ *and* $u_{84,85}(x,y,z,t)]$.

Using (8), the solution of (9) when $[p = 2 - k^2, q = -2, r = -(1-k^2)]$ and the sets of solutions (2)-(7), we get

$$u_{122,123,\dots,133}(x,y,z,t) = a_0 + a_2\left(dn\left(\lambda(x+y+z-ct)\right)\right)^2 + b_2\left(nd\left(\lambda(x+y+z-ct)\right)\right)^2,$$

where a_0, a_2 *and* b_2 are defined in the sets of solutions (2)-(7).

As $k \to 1$, we obtain $[u_{2,3}(x,y,z,t)\ and\ u_{4,5}(x,y,z,t)]$. Also as $k \to 0$, we obtain constant solution.

Using (8), the solution of (9) when $[p = 2 - k^2, q = 2(k^2 - 1), r = -1]$ and the sets of solutions (2)-(7), we get

$$u_{134,135,\dots,145}(x,y,z,t) = a_0 + a_2\left(nd\big(\lambda(x+y+z-ct)\big)\right)^2 + b_2\left(dn\big(\lambda(x+y+z-ct)\big)\right)^2,$$

where $a_0, a_2\ and\ b_2$ are defined in the sets of solutions (2)-(7).

As when $k \to 1$, we obtain $[u_{2,3}(x,y,z,t)\ and\ u_{4,5}(x,y,z,t)]$. Also as $k \to 0$, we obtain constant solution.

Using (8), the solution of (9) when $[p = 2 - k^2, q = 2,\ r = 1 - k^2]$ and the sets of solutions (2)-(7), we get

$$u_{146,147,\dots,157}(x,y,z,t) = a_0 + a_2\left(cs\big(\lambda(x+y+z-ct)\big)\right)^2 + b_2\left(sc\big(\lambda(x+y+z-ct)\big)\right)^2,$$

where $a_0, a_2\ and\ b_2$ are defined in the sets of solutions (2)-(7).

Note that, when $k \to 1$, we obtain

$$u_{158,159}(x,y,z,t) = -6\lambda^2\left(\operatorname{csch}\left(\lambda\left(x+y+z \pm \sqrt{3+4\lambda^2}\,t\right)\right)\right)^2,$$

$$u_{160,161}(x,y,z,t) = -4\lambda^2 - 6\lambda^2\left(\operatorname{csch}\left(\lambda\big(x+y+z \pm \sqrt{3-4\lambda^2}\,t\big)\right)\right)^2,$$ and when $k \to 0$, we obtain

$$u_{162,163}(x,y,z,t) = 4\lambda^2 - 6\lambda^2\left(\cot\left(\lambda\left(x+y+z \pm \sqrt{3+16\lambda^2}\,t\right)\right)\right)^2 - 6\lambda^2\left(\tan\left(\lambda\left(x+y+z \pm \sqrt{3+16\lambda^2}\,t\right)\right)\right)^2,$$

$$u_{164,165}(x,y,z,t) = -12\lambda^2 - 6\lambda^2\left(\cot\left(\lambda\left(x+y+z \pm \sqrt{3-16\lambda^2}\,t\right)\right)\right)^2 - 6\lambda^2\left(\tan\left(\lambda\left(x+y+z \pm \sqrt{3-16\lambda^2}\,t\right)\right)\right)^2,$$

$$u_{166,167}(x,y,z,t) = -2\lambda^2 - 6\lambda^2\left(\tan\left(\lambda\left(x+y+z \pm \sqrt{3+4\lambda^2}t\right)\right)\right)^2,$$

$$u_{168,169}(x,y,z,t) = -6\lambda^2 - 6\lambda^2\left(\tan\left(\lambda\left(x+y+z \pm \sqrt{3-4\lambda^2}\,t\right)\right)\right)^2,$$

$$u_{170,171}(x,y,z,t) = -2\lambda^2 - 6\lambda^2\left(\cot\left(\lambda\left(x+y+z \pm \sqrt{3+4\lambda^2}\,t\right)\right)\right)^2,$$

$$u_{172,173}(x,y,z,t) = -6\lambda^2 - 6\lambda^2\left(\cot\left(\lambda\left(x+y+z \pm \sqrt{3-4\lambda^2}t\right)\right)\right)^2.$$

Using (8), the solution of (9) when $[p = 2 - k^2, q = 2(1 - k^2),\ r = 1]$ and the sets of solutions (2)-(7), we get

$$u_{174,175,\dots,185}(x,y,z,t) = a_0 + a_2\left(sc\big(\lambda(x+y+z-ct)\big)\right)^2 + b_2\left(cs\big(\lambda(x+y+z-ct)\big)\right)^2,$$

where $a_0, a_2\ and\ b_2$ are defined in the sets of solutions (2)-(7).

As $k \to 1$, we obtain$\left[u_{158,159}(x,y,z,t)\ and\ u_{160,161}(x,y,z,t)\right]$. Also as $k \to 0$, we obtain $\left[u_{162,163}(x,y,z,t), u_{164,165}(x,y,z,t), \dots, u_{172,173}(x,y,z,t)\right]$.

Using (8), the solution of (9) when $[p = -1 + 2k^2, q = 2,\ r = -k^2(1 - k^2)]$ and the sets of solutions (2)-(7), we get

$$u_{186,187,\dots,197}(x,y,z,t) = a_0 + a_2\left(ds\big(\lambda(x+y+z-ct)\big)\right)^2 + b_2\left(sd\big(\lambda(x+y+z-ct)\big)\right)^2,$$

where $a_0, a_2\ and\ b_2$ are defined in the sets of solutions (2)-(7).

Note that, when $k \to 1$, we obtain$\left[u_{158,159}(x,y,z,t)\ and\ u_{160,161}(x,y,z,t)\right]$, and when $k \to 0$, we obtain $[u_{54,55}(x,y,z,t)\ and\ u_{56,57}(x,y,z,t)]$.

Using (8), the solution of (9) when $[p = -1 + 2k^2, q = 2k^2(k^2 - 1), r = 1]$ and the sets of solutions (2)-(7), we get

$$u_{198,199,\dots,209}(x,y,z,t) = a_0 + a_2\left(sd\big(\lambda(x+y+z-ct)\big)\right)^2 + b_2\left(ds\big(\lambda(x+y+z-ct)\big)\right)^2,$$

where $a_0, a_2\ and\ b_2$are defined in the sets of solutions (2)-(7).

As $k \to 1$, we obtain $\left[u_{158,159}(x,y,z,t)\ and\ u_{160,161}(x,y,z,t)\right]$. Also as $k \to 0$, we obtain $[u_{54,55}(x,y,z,t)\ and\ u_{56,57}(x,y,z,t)]$.

Conclusion

We successfully obtained exact and explicit analytic solutions to the (3 + 1)-dimensional equation via the Mapping method. Some of these results are in agreement with the results reported by others in the literature, and new results are formally developed in this work. It is shown that the algorithm can be also applied to other (NLPDEs) in mathematical physics. The procedure is simple, direct and constructive with the help of a computer algebra system. The availability of computer systems like Mathematica or Maple facilitates the tedious algebraic calculations.

References

1. Ablowitz, M. J., and Clarkson, P. A.,(1991), Solitons, nonlinear evolution equations and inverse scattering (Vol. 149), Cambridge University press.
2. Ali, A., Iqbal, M. A., and Mohyud-Din, S. T., (2016), Traveling wave solutions of generalized Zakharov–Kuznetsov–Benjamin–Bona–Mahony and simplified modified form of Camassa–Holm equation exp (–φ (η))–Expansion method, Egyptian Journal of Basic and Applied Sciences, 3(2), 134-140.
3. Boz, A., and Bekir, A.,(2008), Application of Exp-function method for (3+ 1)-dimensional nonlinear evolution equations, Computers and Mathematics with Applications, 56(5), 1451-1456.
4. Chen, H-T., Zhang, H-Q.,(2004),New double periodic and multiple soliton solutions of the generalized (2 + 1)-dimensional Boussinesq equation, Chaos, SolitonsFract. 20 765–769.
5. Chen, Y., Yan, Z., Zhang, H-Q.,(2003),New explicit solitary wave solutions for (2 + 1)-dimensional Boussinesq equation and (3 + 1)-dimensional KP equation, Phys. Lett. A 307 107–113.
6. Fan, E., and Zhang, H.,(1998),A note on the homogeneous balance method. Physics Letters A,246(5), 403-406.
7. Fan, E., (2000), Two new applications of the homogeneous balance method. Physics Letters A, 265(5-6), 353-357.
8. Feng, D., He, T., and Lü, J.,(2007),Bifurcations of travelling wave solutions for (2+1)-dimensional Boussinesq-type equation,Applied mathematics and computation,185 (1), 402-414.
9. He, J. H., and Wu, X. H.,(2006),Exp-function method for nonlinear wave equations, Chaos, Solitons and Fractals,30(3), 700-708.
10. He, J. H., and Zhang, L. N.,(2008),Generalized solitary solution and compacton-like solution of the Jaulent–Miodek equations using the Exp-function method, Physics Letters A, 372(7), 1044-1047.
11. He, J. H., and Abdou, M. A. ,(2007),New periodic solutions for nonlinear evolution equations using Exp-function method, Chaos, Solitons and Fractals, 34(5), 1421-1429.
12. Lai, S., Wu, Y.-H., Zhou, Y.,(2008),Some physical structures for the (2 + 1)-dimensional Boussinesq water equation with positive and negative exponents, Computation. Math. Appl. 56 (2) 339–345.

13. Malfliet, W.,(1992),Solitary wave solutions of nonlinear waveequations, AmericanJournal of Physics,60, 650-654.
14. Taşcan, F., and Bekir, A.,(2009),Analytic solutions of the (2+ 1)-dimensional nonlinear evolution equations using the sine–cosine method, Applied Mathematics and Computation, 215(8), 3134-3139.
15. Wazwaz, A. M., (2004),Thetanh method for traveling wave solutions of nonlinear equations, Applied Mathematics and Computation, 154(3), 713-723.
16. Wazwaz, A. M., (2004), A sine-cosine method for handling nonlinear wave equations, Mathematical and Computer modelling, 40(5-6), 499-508.
17. Wazwaz, A. M.,(2004),The sine–cosine method for obtaining solutions with compact and noncom pact structures, Applied Mathematics and Computation, 159(2), 559-576
18. Wazwaz, A. M.,(2005),Thetanh method and the sine–cosine method for solving the KP-MEW equation, International Journal of Computer Mathematics, 82(2), 235-246.
19. Wazwaz, A. M., (2007), The extended tanh method for new solitons solutions for many forms of the fifth-order KdV equations, Applied Mathematics and Computation, 184(2), 1002-1014.
20. Wazwaz, A. M.,(2007),The extended tanh method for abundant solitary wave solutions of nonlinear wave equations, Applied Mathematics and Computation, 187(2), 1131-1142.
21. Wazwaz, A. M.,(2007),Newsolitons and kinks solutions to the Sharma–Tasso–Olver equation, Applied Mathematics and Computation, 188(2), 1205-1213.
22. Wazwaz, A. M.,(2007),Multiple-soliton solutions for the KP equation by Hirota's bilinear method and by the tanh–coth method,Applied Mathematics and Computation, 190(1), 633-640.
23. Wazwaz, A. M.,(2008),The Hirota's direct method for multiple-soliton solutions for three model equations of shallow water waves, Applied Mathematics and Computation, 201(12), 489-503.

الحلول التحليلية للنموذج الجديد لمعادلة بوسنيك ذات الأبعاد الثلاثة

مبارك عبود حامد القفيل، محمد سالم أحمد العمري و بثينة مهدي محمد صالح عون
[1]قسم الرياضيات، كلية التربية - عدن، جامعة عدن
DOI: https://doi.org/10.47372/uajnas.2019.n2.a17

الملخص

في هذا البحث قمنا بتطبيق طريقة المابينج لإيجاد حلول معادلة بوسنيك. حيث حصلنا على العديد من العائلات الجديدة من الحلول الجديدة الدقيقة للموجة والمعبرة عنها بواسطة الدوال الزائدية، الدوال المثلثية، الدوال الكسرية والدوال الجاكوبية. إذ تًعد طريقة المابينج أداة قوية لحل العديد من المعادلات التفاضلية الجزئية غير الخطية في الرياضيات وفي العلوم الفيزيائية.

الكلمات المفتاحية: طريقة المابينج، الحلول الدقيقة، معادلة بوسينك (3+1).