On certain a generalized $N_{|m}$–Recurrent Finsler space

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Abstract

A Finsler space F_n for which the normal projective curvature tensor N^i_{jkh} satisfies $N^i_{jkh|m} = \lambda_m N^i_{jkh} + \mu_m(\delta^i_h g_{jk} - \delta^i_k g_{jh})$, $N^i_{jkh} \neq 0$, where λ_m and μ_m are non-zero covariant vectors field, will be called *a generalized $N_{|m}$– recurrent space*. The curvature vector H_k, the curvature scalar H and Ricci tensor N_{jk} are non-vanishing. When the generalized $N_{|m}$– recurrent space is affinely connected space and under certain conditions, we obtain various results. Also, in generalized $N_{|m}$– recurrent space, Weyl's projective curvature tensoris a generalized recurrent tensor.

Keywords: Generalized $N_{|m}$-Recurrent Space, Generalized Recurrent Tensor, Generalized $N_{|m}$-Recurrent Affinely Connected Space, Weyl's projective curvature recurrent tensor.

1.Introduction

K.Yano [20] defined the normal projective connection Π^i_{jk} by

(1.1) $\Pi^i_{jk} = G^i_{jk} - \frac{1}{n+1} y^i G^r_{jkr}$.

R.B.Misra and F.M.Meher [12] considered a space equipped with normal projective connection Π^i_{jk} whose curvature tensor $N^i_{jkh}$is recurrent with respect to normal projective connection Π^i_{jk} and they called it *RNP-Finsler space*. P.N. Pandey and V.J. Diwivedi [16]studied*RNP*-Finsler space and obtained many identities in *RNP*-Finsler space, most of these identities are also true in a recurrent Finsler space with respect to Berwald's connection coefficientsG^i_{jk}. F. Y. A. Qasem [17] obtained several results concerningthe normal projective curvature tensorN^i_{jkh} in such space.

Let us consider a set of quantitiesg_{ij} defined by [18]

(1.2) $g_{ij}(x,y) = \frac{1}{2}\dot{\partial}_i\dot{\partial}_j F^2(x,y)$.

The tensor$g_{ij}(x,y)$is positively homogeneous of degree zero in $y^i$and symmetric in i and j. According to Euler's theorem on homogeneous functions,the vectors$y_i$and y^isatisfythe following relations [18]

(1.3) $a)\ y_i y^i = F^2, b)\ g_{ij} = \dot{\partial}_i y_j = \dot{\partial}_j y_i$ and $c)\ g_{ij} y^i = y_j$.

Cartan's covariant derivative of the metric function F, vector$y^i$and the metric tensor g_{ij}vanish identically, i.e. [18]

(1.4)$a)\ F_{|k} = 0, b)\ y^i_{|k} = 0andc)\ g_{ij|k} = 0$.

A Finsler space whose connection parameter$G^i_{jk}$is independent ofy^i is called an *affinely connected space*[1]. Thus,an affinely connected space is characterized by one of the equivalent equations

(1.5) $a)\ G^i_{jkh} = 0$and$b)\ C_{ijk|h} = 0$.

The connection parameter$\Gamma^{*i}_{kh}$of Cartan and $G^i_{jk}$of Berwald coincide in affinely connected space and they are independent of the direction argument, i.e. [18]

(1.6) $a)\dot{\partial}_j G^i_{kh} = 0andb)\ \dot{\partial}_j \Gamma^{*i}_{kh} = 0$.

Cartan's connection parameterΓ^{*i}_{kh} coincides with Berwald's connection parameter$G^i_{kh}$for a Landsberg space, which is characterized by [18]

(1.7) $y_r G^r_{jkh} = -2C_{jkh|r} y^r = -2P_{jkh} = 0.$

Various authors denote the tensor $C_{jkh|r} y^r$ by P_{jkh} F. Ikeda[2],H. Izumi **(**[4]-[7]), H. Izumiand M. Toshida [8],M. Matsumoto [10] and H. Wosoughi [20].Since the equations (1.5a) and (1.6a) imply(1.7), an affinely connected space is necessarily Landsberg space[1]. However, a Landsberg space need not be an affinely connected space.

Cartan's covariant derivative of an arbitrary tensor T^i_h with respect to x^k is given by

(1.8) $a)\ \dot\partial_j\left(T^i_{h|k}\right) - (\dot\partial_j T^i_h)_{|k} = T^r_h(\dot\partial_j \Gamma^{*i}_{kr}) - T^i_r(\dot\partial_j \Gamma^{*r}_{kh}) - \left(\dot\partial_r T^i_h\right)P^r_{kj},$

where

$$b)\ P^r_{kj} = \left(\dot\partial_j \Gamma^{*r}_{hk}\right) y^h = \Gamma^{*r}_{jhk} y^h$$

and

$$c)\ P^r_{kj} = g^{ir} P_{rkh}.$$

The tensor H^i_{jkh} is called *Berwaldcurvature tensor*,it is positively homogeneous of degree zero in y^i and skew-symmetric in its last two lower indices which defined by [18]

$H^i_{jkh} := \partial_h G^i_{jk} + G^r_{jk} G^i_{rh} + G^i_{rk} G^r_j - h/k.$

In view of Euler's theorem on homogeneous functions, we have the following relations [18]

(1.9) $a)\ \dot\partial_j H^i_{kh} = H^i_{jkh}, b)\ H^i_{jkh} y^j = H^i_{kh}, c)\ H_{ijkh} \coloneqq g_{jr} H^r_{ikh},$

$$d)\ H^i_{kh} y^k = H^i_h, e)\ H^i_{kh} = \dot\partial_k H^i_h, f)\ H_{jk} = H^r_{jkr},$$

$g)\ H_k = H^r_{kr}$ and $h)\ H = \frac{1}{n-1} H^r_r.$

The tensor $H_{jk.h}$ defined by

(1.10) $H_{jk.h} = g_{ik} H^i_{jh}.$

2.Normal Projective Curvature Tensor

P.N. Pandey ([13] - [15]) obtained a relation between the normal projective curvature tensor N^i_{jkh} and Berwald curvature tenser H^i_{jkh} as follows:

(2.1) $N^i_{jkh} = H^i_{jkh} - \frac{1}{n+1} y^i \dot\partial_j H^r_{rkh}.$

The normal projective curvature tensor N^i_{jkh} is homogeneous of degree zero in y^i.

Contracting the indices i and j in (2.1) and using the fact that the tensor H^r_{rkh} is positively homogeneous of degree zero in y^i, we get

(2.2) $N^r_{rkh} = H^r_{rkh}.$

Transvecting (2.1) by y^j and using (1.9b), we get

(2.3) $N^i_{jkh} y^j = H^i_{kh}.$

The projective curvature tensor W^i_{jkh} and the normal projective curvature tensor N^i_{jkh} are connected [9] by

(2.4) $a)\ W^i_{jkh} = N^i_{jkh} + (\delta^i_k M_{hj} - M_{kh}\delta^i_j - k|h)$,

where

$$b)\ M_{kh} := -\frac{1}{n^2-1}(nN_{kh} + N_{hk})$$

and

$$c) N_{jk} := N^r_{jkr}.$$

The projective curvature tensor W^i_{jkh} satisfies the following [18]:

(2.5) $a)\ W^i_{jkh} y^j = W^i_{kh}$ and $b)\ W^i_{kh} y^k = W^i_h.$

A Finsler space is called *a recurrent Finsler space* if it's normal projective curvature tensor N^i_{jkh} satisfies ([11], [14], [19])

(2.6) $N^i_{jkh|m} = \lambda_m N^i_{jkh}, \qquad N^i_{jkh} \neq 0,$

where λ_m is non-zero covariant vector field.

3.Generalized Recurrent Space

Let us consider a Finsler space$F_n$for which the normal projective curvature tensor N^i_{jkh}satisfies the condition

(3.1)$N^i_{jkh|m} = \lambda_m N^i_{jkh} + \mu_m\left(\delta^i_h g_{jk} - \delta^i_k g_{jh}\right), N^i_{jkh} \neq 0,$

where$\lambda_m$and μ_m are non–zero covariant vectors field,such space will be called *a generalized $N_{|m}$– recurrent space* and the tenser will be called *generalized $N_{|m}$– recurrent tensor.*

Remark 3.1. Any curvature tensor which satisfies similar to the condition (3.1) will be called*generalized recurrent tensor*.

Contracting the indices i and j in (3.1) and using (2.2), we get

(3.2) $H^r_{rkh|m} = \lambda_m H^r_{rkh}.$

Thus, the followingtheorem

Theorem 3.1.*Ingeneralized $N_{|m}$– recurrent* space, *Cartans covariantderivative of thetensorH^r_{rkh} behaves as recurrent.*

Transvecting (3.1) by y^j, using (1.4b), (2.3) and(1.3c),we get

(3.3) $H^i_{kh|m} = \lambda_m H^i_{kh} + \mu_m\left(\delta^i_h y_k - \delta^i_k y_h\right).$

Transvecting (3.3) by y^k, using (1.4b), (1.9d) and (1.3a), we get

(3.4) $H^i_{h|m} = \lambda_m H^i_h + \mu_m\left(\delta^i_h F^2 - y^i y_h\right).$

Thus, the followingtheorem

Theorem 3.2.*Ingeneralized $N_{|m}$–recurrentspace, Cartans covariantderivative of the h(v)–torsion tensor H^i_{kh} and the deviation tensor $H^i_h$are given by (3.3) and (3.4), respectively.*

Contracting the indices i and h in (3.3)and using (1.9g), we get

(3.5) $H_{k|m} = \lambda_m H_k + (n-1)\mu_m y_k.$

Contracting the indices i and h in (3.4)and using (1.9h), we get

(3.6) $H_{|m} = \lambda_m H + \mu_m F^2.$

Contracting the indices i and h in (3.1)and using (2.4c), we get

(3.7)$N_{jk|m} = \lambda_m N_{jk} + (n-1)\mu_m g_{jk}.$

Thus, the followingtheorem

Theorem 3.3.*The curvature vector H_k,the curvaturescalar H and Ricci tensorN_{jk} of generalized $N_{|m}$– recurrentspace arenon – vanishing.*

Differentiating (3.2) partially with respect toy^j, we get

(3.8) $\dot{\partial}_j\left(H^r_{rkh|m}\right) = \left(\dot{\partial}_j \lambda_m\right) H^r_{rkh} + \lambda_m \dot{\partial}_j H^r_{rkh}.$

Differentiating (2.1) covariantly with respect to x^m in the sense of Cartan and using (1.4b), we get

(3.9) $N^i_{jkh|m} = H^i_{jkh|m} - \frac{1}{n+1} y^i (\dot{\partial}_j H^r_{rkh})_{|m}.$

Using commutation formula exhibited by (1.8a) forH^r_{rkh} in (3.9), using (3.1) and (3.8), we get

(3.10) $\lambda_m N^i_{jkh} + \mu_m\left(\delta^i_h g_{jk} - \delta^i_k g_{jh}\right) = H^i_{jkh|m} - \frac{1}{n+1} y^i\{(\dot{\partial}_j \lambda_m) H^r_{rkh}$
$+\lambda_m \dot{\partial}_j H^r_{rkh} + H^r_{rsh}\left(\dot{\partial}_j \Gamma^{*s}_{km}\right) + H^r_{rks}\left(\dot{\partial}_j \Gamma^{*s}_{hm}\right) + \left(\dot{\partial}_s H^r_{rkh}\right) P^s_{jm}\}.$

Using (2.1) in (3.10), we get

(3.11) $\lambda_m H^i_{jkh} + \mu_m\left(\delta^i_h g_{jk} - \delta^i_k g_{jh}\right) = H^i_{jkh|m} - \frac{1}{n+1} y^i\{\left(\dot{\partial}_j \lambda_m\right) H^r_{rkh}$
$+H^r_{rsh}\left(\dot{\partial}_j \Gamma^{*s}_{km}\right) + H^r_{rks}\left(\dot{\partial}_j \Gamma^{*s}_{hm}\right) + \left(\dot{\partial}_s H^r_{rkh}\right) P^s_{mj}\}.$

This shows that

$$H^i_{jkh|m} = \lambda_m H^i_{jkh} + \mu_m\left(\delta^i_h g_{jk} - \delta^i_k g_{jh}\right)$$

if and only if

(3.12)$\left(\dot{\partial}_j \lambda_m\right) H^r_{rkh} + H^r_{rsh}\left(\dot{\partial}_j \Gamma^{*s}_{km}\right) + H^r_{rks}\left(\dot{\partial}_j \Gamma^{*s}_{hm}\right) + \left(\dot{\partial}_s H^r_{rkh}\right) P^s_{jm} = 0.$

Thus, the followingtheorem

Theorem 3.4. *In generalized $N_{|m}$-recurrent space, Berwaldcurvature tensor H^i_{jkh} is generalized recurrent if and only if(3.12)holds.*

Contracting the indices i and h in (3.11) and using (1.9f), we get

(3.13) $$\lambda_m H_{jk} + \mu_m (n-1) g_{jk} = H_{jk|m} - \frac{1}{n+1} y^t \{(\dot{\partial}_j \lambda_m) H^r_{rkt} + H^r_{rst}(\dot{\partial}_j \Gamma^{*s}_{km}) + H^r_{rks}(\dot{\partial}_j \Gamma^{*s}_{tm}) + (\dot{\partial}_s H^r_{rkt}) P^s_{jm}\}.$$

This shows that

$$H_{jk|m} = \lambda_m H_{jk} + \mu_m (n-1) g_{jk}.$$

if and only if

(3.14) $$y^t \{(\dot{\partial}_j \lambda_m) H^r_{rkt} + H^r_{rst}(\dot{\partial}_j \Gamma^{*s}_{km}) + H^r_{rks}(\dot{\partial}_j \Gamma^{*s}_{tm}) + (\dot{\partial}_s H^r_{rkt}) P^s_{mj}\} = 0.$$

Thus, the following theorem

Theorem 3.5. *In generalized $N_{|m}$−recurrent space, Ricci tensor $H_{jk}$is non – vanishing if and only if(3.14)holds.*

Also, (3.11) can be written as

(3.15) $$H^b_{jkh|m} - \lambda_m H^b_{jkh} - \mu_m (\delta^b_h g_{jk} - \delta^b_k g_{jh}) = \frac{y^b}{n+1} \{(\dot{\partial}_j \lambda_m) H^r_{rkh} + H^r_{rsh}(\dot{\partial}_j \Gamma^{*s}_{km}) + H^r_{rks}(\dot{\partial}_j \Gamma^{*s}_{hm}) + (\dot{\partial}_s H^r_{rkh}) P^s_{jm}\}.$$

Transvecting (3.15)by$y_b$and using (1.3a), we get

(3.16) $$\frac{y_b}{F^2} \{H^b_{jkh|m} - \lambda_m H^b_{jkh} - \mu_m (\delta^b_h g_{jk} - \delta^b_k g_{jh})\} = \frac{1}{n+1} \{(\dot{\partial}_j \lambda_m) H^r_{rkh} + H^r_{rsh}(\dot{\partial}_j \Gamma^{*s}_{km}) + H^r_{rks}(\dot{\partial}_j \Gamma^{*s}_{hm}) + (\dot{\partial}_s H^r_{rkh}) P^s_{jm}\}.$$

From (3.15) and (3.16), we get

(3.17) $$H^i_{jkh|m} - \lambda_m H^i_{jkh} - \mu_m (\delta^i_h g_{jk} - \delta^i_k g_{jh}) = \frac{y_b y^i}{F^2} \{H^b_{jkh|m} - \lambda_m H^b_{jkh} - \mu_m (\delta^b_h g_{jk} - \delta^b_k g_{jh})\}.$$

Thus, the following theorem

Theorem 3.6. *In generalized $N_{|m}$ – recurrent space, the curvature tensor $H^i_{jkh}$is generalized recurrentif and only if$y_b\{H^b_{jkh|m} - \lambda_m H^b_{jkh} - \mu_m (\delta^b_h g_{jk} - \delta^b_k g_{jh})\} = 0$holds.*

Transvecting (3.3)byg_{si}, using (1.10) and (1.4c), we get

(3.18) $$H_{ks.h|m} = \lambda_m H_{ks.h} + \mu_m (g_{sh} y_k - g_{sk} y_h).$$

Thus, the following theorem

Theorem 3.7. *In generalized $N_{|m}$ – recurrentspace,Cartans covariantderivative of theassociate tensor$H_{ks.h}$of the h(v)-torsion tensor$H^i_{kh}$is given by (3.18).*

Transvecting (3.11)byg_{ti}, using (1.9c), (1.3c) and (1.4c), we get

(3.19) $$\lambda_m H_{jtkh} + \mu_m (g_{th} g_{jk} - g_{tk} g_{jh}) = H_{jtkh|m} - \frac{1}{n+1} y_t \{(\dot{\partial}_j \lambda_m) H^r_{rkh} + H^r_{rsh}(\dot{\partial}_j \Gamma^{*s}_{km}) + H^r_{rks}(\dot{\partial}_j \Gamma^{*s}_{hm}) + (\dot{\partial}_s H^r_{rkh}) P^s_{mj}\}.$$

This shows that

(3.20) $$H_{jtkh|m} = \lambda_m H_{jtkh} + \mu_m (g_{th} g_{jk} - g_{tk} g_{jh})$$

if and only if

(3.21) $$(\dot{\partial}_j \lambda_m) H^r_{rkh} + H^r_{rsh}(\dot{\partial}_j \Gamma^{*s}_{km}) + H^r_{rks}(\dot{\partial}_j \Gamma^{*s}_{hm}) + (\dot{\partial}_s H^r_{rkh}) P^s_{mj} = 0.$$

Thus, the followingtheorem

Theorem 3.8. *In generalized $N_{|m}$ – recurrentspace,theassociatetensor H_{jskh} of the curvature tensor$H^i_{jkh}$is given by (3.20) if and only if(3.21)holds.*

Remark 3.2. Ifthegeneralized $N_{|\overline{m}}$ recurrentspaceis affinely connected space, so the new space will be *called generalized $N_{|m}$−recurrent space affinely connected space.* It will be sufficient to call the curvature tensor which satisfies this space by generalized recurrent.

Let us consider generalized $N_{|m}$–recurrentaffinely connected space.

In view of (1.8c), (1.7) and if $\dot{\partial}_j \lambda_m = 0$,(3.11) becomes

(3.22)$H^i_{jkh|m} = \lambda_m H^i_{jkh} + \mu_m\left(\delta^i_h g_{jk} - \delta^i_k g_{jh}\right)$.

Thus, the following theorem

Theorem 3.9.*In the generalized $N_{|m}$– recurrentaffinely connected space,ifthe directional derivative of covariant vector field vanish, then thecurvature tensorH^i_{jkh} is generalized recurrent.*

In view of (1.8c), (1.7)and if $\dot{\partial}_j \lambda_m = 0$,(3.19) becomes

(3.23) $H_{jtkh|m} = \lambda_m H_{jtkh} + \mu_m\left(g_{th} g_{jk} - g_{tk} g_{jh}\right)$.

Thus, the followingtheorem

Theorem 3.10.*In the generalized $N_{|m}$ –recurrentaffinely connected space,if the directional derivative of covariant vector field vanish, then theassociatetensor$H_{jskh}$of thecurvature tensor $H^i_{jkh}$is generalized recurrent.*

In view of (1.8c), (1.7) and if $\dot{\partial}_j \lambda_m = 0$,(3.13) becomes

(3.24)$H_{jk|m} = \lambda_m H_{jk} + \mu_m(n-1) g_{jk}$

Thus, the followingtheorem

Theorem 3.11.*In the generalized $N_{|m}$– recurrentaffinely connected space, if the directional derivative of covariant vector field vanish, then the H –Ricci tensor$H_{jk}$is non-vanishing.*

Remark 3.3. Anaffinely connected space is necessarilyLandsberg space. However, Landsberg space need not bean affinely connected space. Hence, any result obtained in affinely connected spaceare satisfiesLandsberg space.

4.Weyl'sProjective CurvatureGeneralized$N_{|m}$ – RecurrentSpace

Letus consider a Finsler space$F_n$for which the normal projective curvature tensor N^i_{jkh}satisfies the condition (3.1).

Differentiating (2.4b) covariantly with respect to x^m in the sense of Cartan, we get

(4.1) $M_{kh|m} = -\frac{1}{n^2-1}\left(nN_{kh|m} + N_{hk|m}\right)$.

Using (3.7) in (4.1), we get

(4.2) $M_{kh|m} = \lambda_m \left\{-\frac{1}{n^2-1}(nN_{kh} + N_{hk})\right\} - \frac{2}{n+1}\mu_m g_{kh}$.

Using (2.4b) in (4.2), we get

(4.3) $M_{kh|m} = \lambda_m M_{kh} - \frac{2}{n+1}\mu_m g_{kh}$.

Thus, the following theorem

Theorem 4.1.*In generalized $N_{|m}$– recurrentspace,Cartan derivative ofthetensor $M_{kh}$is given by (4.3).*

Differentiating (2.4a) covariantly with respect to x^m in the sense of Cartan, we get

(4.4)$W^i_{jkh|m} = N^i_{jkh|m} + (\delta^i_k M_{hj|m} - M_{kh|m}\delta^i_j - k/h)$.

Using (3.1)and (4.3)in (4.4), we get

(4.5)$W^i_{jkh|m} = \lambda_m\{N^i_{jkh} + (\delta^i_k M_{hj} - M_{kh}\delta^i_j - k/h\} + \mu_m(\delta^i_h g_{jk} - \delta^i_j g_{kh})$.

Using (2.4a) in (4.5), we get

(4.6)$W^i_{jkh|m} = \lambda_m W^i_{jkh} + \mu_m(\delta^i_h g_{jk} - \delta^i_j g_{kh})$.

Thus, the following theorem

Theorem 4.2.*In generalized $N_{|m}$–recurrent space,the projective curvaturetensorW^i_{jkh}isgeneralized recurrent.*

Transvecting (4.6)byy^j, using (2.5a), (1.4b) and (1.3c),we get

(4.7)$W^i_{kh|m} = \lambda_m W^i_{kh} + \mu_m(\delta^i_h y_k - y^i g_{kh})$.

Transvecting (4.7)byy^k, using (2.5b), (1.4b) and (1.3a), we get

(4.8)$W^i_{h|m} = \lambda_m W^i_h + \mu_m(\delta^i_h F^2 - y^i y_h)$.

Thus, the followingtheorem

Theorem 4.2.*In generalized $N_{|m}$ – recurrentspace,Cartan derivativeofthe projective torsiontensorW^i_{kh} andtheprojective deviation tensorW^i_hgiven by (4.7) and (4.8),respectively.*

Now, we know that Finsler spaceF_n, in general, is not generalized $N_{|m}$– recurrent spaceif the tensor M_{kh} of Finsler space $F_n$is given by (4.3). But ifthe projective curvature tensorW^i_{jkh} is generalized recurrenttensor, our space is necessarilygeneralized $N_{|m}$-recurrent space and this may be seen as follows:

Let us consider a Finsler space $F_n$in which the projective curvature tensor$W^i_{jkh}$and the tensor $M_{kh}$are generalized recurrenttensors.

Differentiating (2.4a) covariantly with respect to x^m in the sense of Cartan, we get

(4.9)$N^i_{jkh|m} = W^i_{jkh|m} - \left(\delta^i_k M_{hj|m} - M_{kh|m}\delta^i_j - k\middle|h\right)$.

Using (4.3), (4.6) and the properties δ^i_k in (4.9), we get

(4.10)$N^i_{jkh|m} = \lambda_m\left\{W^i_{jkh} - \left(\delta^i_k M_{hj} - M_{kh}\delta^i_j - k\middle|h\right)\right\} + \mu_m\left(\delta^i_h g_{jk} - \delta^i_j g_{kh}\right)$.

Using (2.4a) in (4.10), we get

(4.11)$N^i_{jkh|m} = \lambda_m N^i_{jkh} + \mu_m\left(\delta^i_h g_{jk} - \delta^i_j g_{kh}\right)$.

Thus, the followingtheorem

Theorem 4.3.*InFinsler space F_n, if theprojective curvaturetensorW^i_{jkh} and thetensor $M_{kh}$are generalized recurrenttensors, then the space considered is necessarily generalized $N_{|m}$-recurrent space.*

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حول تعميم فضاء $N_{|m}$-أحادي المعاودة

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الملخص

فضاء فنسلر الذي يحقق فيه الموتر التقوسي الاسقاطي العادي N^i_{jkh} الشرط الأتية:

$$N^i_{jkh|m} = \lambda_m N^i_{jkh} + \mu_m\left(\delta^i_h g_{jk} - \delta^i_k g_{jh}\right), \quad N^i_{jkh} \neq 0,$$

حيث λ_m و μ_m هي متجهات متحدة الاختلاف لا تساوي الصفر، وتم تسمية هذا الفضاء الذي يحقق الشرط أعلاه بتعميم فضاء $N_{|m}$ -أحادي المعاودة.

كما أُثبِت أن المتجه التقوسيH_k، الثابت التقوسي H، وموتر ريتشي N_{kh}، كلها لا تنتهي في تعميم فضاء $N_{|m}$ -أحادي المعاودة. وكذلك لكي يكون موتر ريتشي بمفهوم كرتان H_{jk} غير منتهٍ وذلك عندما يكون تعميم فضاء $N_{|m}$ -أحادي المعاودة هو فضاء أفينلي وتكون المشتقة الاتجاهية بالنسبة للإحداثي الاتجاهي لمتجهات متحدة الاختلاف منتهية. وفي تعميم فضاء $N_{|m}$ -أحادي المعاودة أًثبِت ان موتر ويلي التقوسي الإسقاطي W^i_{jkh} و هو معمم أحادي المعاودة.

الكلمات المفتاحية: تعميم فضاء $N_{|m}$ -أحادي المعاودة، تعميم موتر حادي المعاودة، تعميم فضاء افينلي$N_{|m}$ - أحادي المعاودة، تعميم موتر ويلي التقوسي الإسقاطي.